Numerical Computation of the Jordan Canonical Form

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One of the main research subjects of T. Y. Li has been matrix computation, especially the algebraic eigenvalue problem. This talk will present a two-staged algorithm, jointly developed with T. Y. Li, for accurate computation of the Jordan Canonical Form and multiple eigenvalues of general matrices.

The Jordan Canonical Form (JCF) is of fundamental importance in matrix analysis, ODE, control theory and beyond. However, it is often mentioned in the literature that JCF may be intractable in numerical computation because it is infinitely sensitive to data perturbations and round-off error. In a larger context, computing JCF is a typical ill-posed problem characterized by Hadamard. The proposed method is based on a novel reformulation of the JCF problem in a least squares setting with a structural constraint. This regularization mechanism removes ill-posedness and problem sensitivity, making it possible for JCF to be calculated accurately. The algorithms consists of an opening stage that determines the JCF structure by computing the minimal polynomial via approximate rank-revealing, and a concluding stage that calculates the Jordan decomposition by solving the reformulated least squares problem. We shall present a prototype software package and numerical results that demonstrate the effectiveness of the algorithm.

Ill-posed problems arise frequently in physical sciences. Likewise, many basic problems in numerical analysis are also ill-posed, such as matrix rank-revealing, multiple zeros, the polynomial GCD, polynomial factorization, multiplicity structure of polynomial systems, along with matrix JCF. This talk will also discuss similar regularization strategies for other ill-posed problems.
