Sensitivity of Natural Measure Calculation

Tim Sauer
George Mason University
tsauer@gmu.edu

The calculation of measures invariant under nonlinear systems is a key problem in dynamical systems research. Ulam conjectured a Frobenius-Perron approach to the problem for one-dimensional maps. Li gave a computation-oriented proof of the conjecture in some of his earliest work [Li 1976].

In computer simulations of deterministic dynamical systems, floating point rounding errors and other truncation errors contaminate results. In particular, chaotic dynamics cause exponential divergence of computed trajectories from their correct versions, but shadowing theory has helped to alleviate the problem under hyperbolic and near-hyperbolic conditions. In particular, one may ask how chaotic dynamics affects computation of natural measure in the presence of rounding and truncation errors.

We report on some effects of computations using IEEE finite precision arithmetic on inference of natural measure of chaotic attractors. First, the effects on familiar examples are studied. Secondly, examples are constructed whose dynamics provide significant obstructions to accurate finite precision calculation of measure.

[Li 1976] T.-Y. Li, Finite approximation for the Frobenius-Perron operator, a solution to Ulam's conjecture. J. Approx. Theory 17, 177-186 (1976).