Sums of Squares Relaxation of Polynomial Optimization Problems

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Given a real-valued continuous function (objective function) defined on the n-dimensional Euclidean space and a subset of the space (a constraint set), we consider a problem of finding a point in the constraint set that minimizes the objective function. This is a fundamental problem in the field of mathematical programming, and numerical computation of such a point is sometimes called global optimization. Without any additional assumptions, this problem is too difficult to analyze and to design efficient numerical methods. A polynomial optimization problem (POP) is a problem of minimizing a (real valued multivariate) polynomial objective function over a constraint set described by a finite set of polynomial inequalities. The POP covers many important nonconvex optimization problems such as 0-1 integer programs and quadratic programs. It serves as a unified and general mathematical model for global optimization. If only polynomials are considered in global optimization, many profound results from algebra can be used to analyze the problem and introduce efficient numerical methods. In recent years, sums of squares (SOS) relaxation for POPs has been proposed and studied extensively. The fundamental idea behind the SOS relaxation lies on a simple fact that a sum of squares of finitely many polynomials is a nonnegative polynomial (but the converse is not true in general). A hierarchy of convex relaxation problems with increasing sizes is constructed in the SOS relaxation of a POP. Each relaxation problem can be numerically solved as a semidefinite programming problem. Under a moderate assumption, a convex relaxation problem with a finite size in the hierarchy attains the exact optimal value of the original POP. The main purpose of this talk it to present the effectiveness of the SOS relaxation of POPs in theory and practice.
