

Approximations of Absolutely Continuous Invariant Measures

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Let Ω be a bounded region of R^d , and let $S : \Omega \rightarrow \Omega$ be a non-singular transformation such that the corresponding *Frobenius-Perron operator* $P : L^1(\Omega) \rightarrow L^1(\Omega)$, which is defined by

$$\int_A P f dm = \int_{S^{-1}(A)} f dm,$$

where m is the Lebesgue measure of R^d , has a fixed density f^* , which is the density of an absolutely continuous invariant measure for S .

Fixed densities of Frobenius-Perron operators determine the statistical properties of chaotic dynamical systems, and their computation is important in various physical sciences such as statistical physics and mathematical biology. A well known numerical scheme is Ulam's method based on piecewise constant approximations, in which the Frobenius-Perron operator is approximated by a finite dimensional *Markov operator* whose representation under the canonical density basis is a stochastic matrix

$$P_h = [p_{ij}], \quad p_{ij} = \frac{m(\Omega_i \cap S^{-1}(\Omega_j))}{m(\Omega_i)},$$

where $\{\Omega_i\}, i = 1, 2, \dots, l$ give a partition of Ω .

Ulam conjectured that if the Frobenius-Perron operator has a fixed density, then his numerical method is convergent. This conjecture was first proved by Tien Yien Li in 1976 for the Lasota-Yorke class of piecewise C^2 and stretching interval mappings, which initiated the research on the numerical analysis of Frobenius-Perron operators.

In this talk I will survey some of the developments on finite approximations of Frobenius-Perron operators in the past 15 years.
