

How to solve a differential equation, when dsolve doesn't work.

Predator-prey model

```
> dsolve( {diff(x(t),t) = x(t) -0.4*x(t)*y(t),
           diff(y(t),t) = 0.5*x(t)*y(t)-0.1*y(t) }, {x(t),y(t)});
```

$$\left[\left\{ y(t) = 0 \right\}, \left\{ x(t) = _C1 e^{t} \right\}, \left[\left[y(t) = \text{RootOf} \left(- \left(\int \frac{10}{_a \left(\ln \left(-\text{LambertW} \left(- \frac{e^{-1} (e^{-a})^4 (e^{-C1})^2}{_a^{10}} \right) \right) - 4_a + 10 \ln(_a) - 2_C1}{d_a} \right) + t + _C2 \right) \right], \left\{ x(t) = \frac{1}{5} \frac{10 \left(\frac{d}{dt} y(t) \right) + y(t)}{y(t)} \right\} \right] \right] \right] \quad (1)$$

```
> # Now, find an approximate solution instead
restart:
read("d:/475/RungeKutta.mpl"):

> # To apply the program, prepare input
f := Vector(2): # the RHS functions
f[1] := (t,x,y) -> x - 0.1*x*y:
f[2] := (t,x,y) -> 0.02*x*y-0.5*y:

> f;
```

$$\begin{bmatrix} (t, x, y) \rightarrow x - 0.1 x y \\ (t, x, y) \rightarrow 0.02 x y - 0.5 y \end{bmatrix} \quad (2)$$

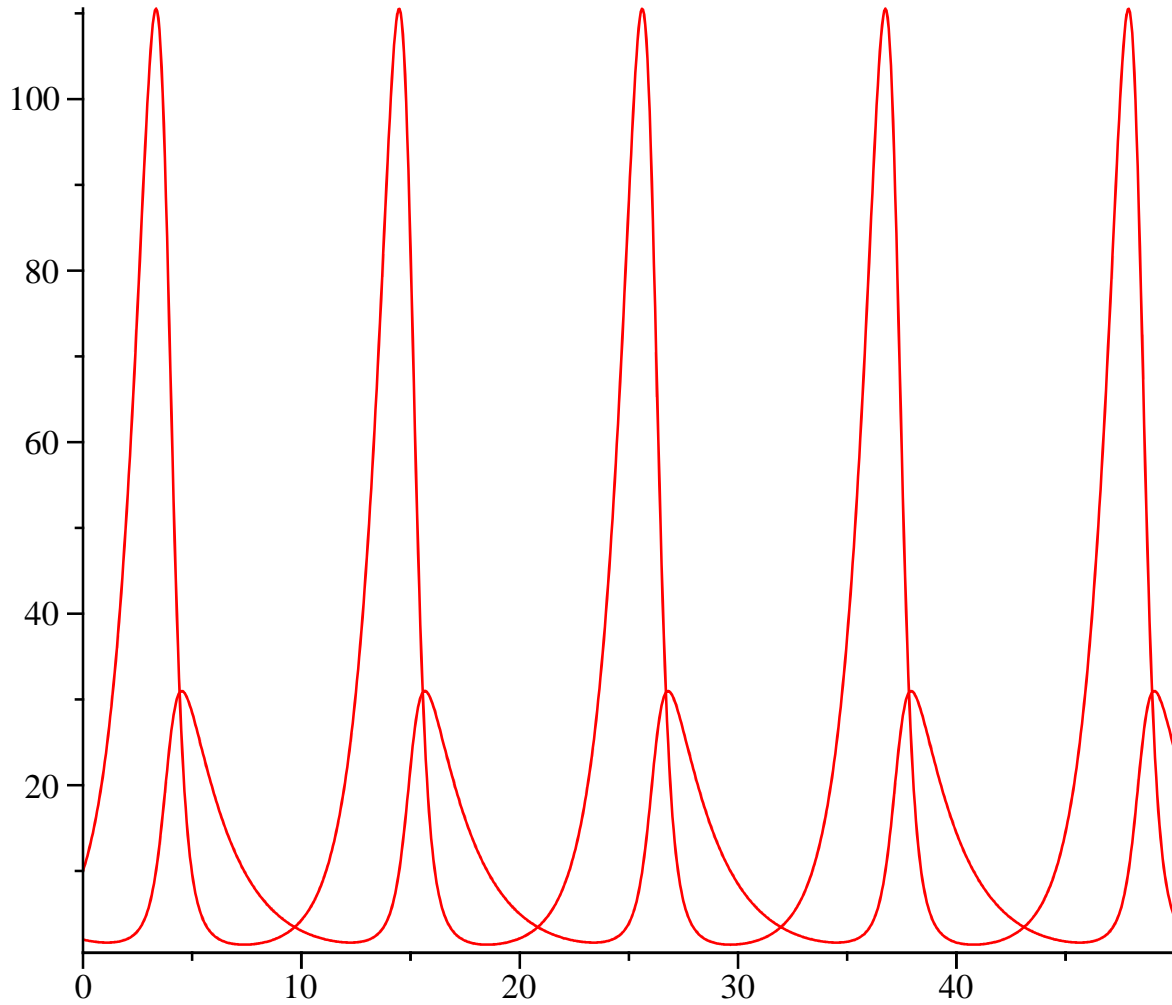
```
> a, b := 0, 50: # the time interval
> c := Vector([10,2]): # initial values
> m := 1000:
> t, y := RungeKutta(f,a,b,c,m);
```

(3)

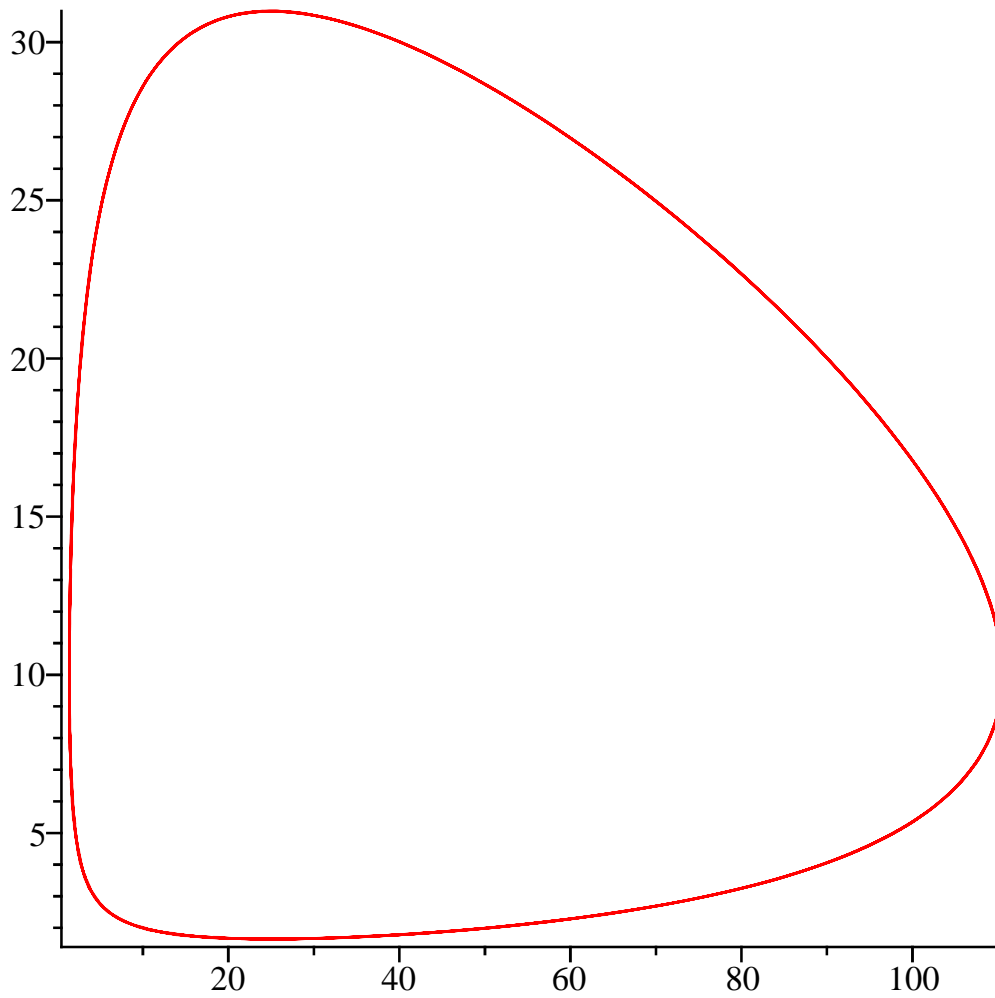
$t, y := \left[\begin{array}{l} 1 \dots 1001 \text{ Vector}_{\text{column}} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran_order} \end{array} \right], x$

(3)

```
> plot1 := plot( [seq([t[k],y[1][k]],k=1..m)] ):  
> plot2 := plot( [seq([t[k],y[2][k]],k=1..m)] ):  
> plots[display]({plot1,plot2});
```



```
> plot( [seq([y[1][k],y[2][k]],k=1..m)] );
```



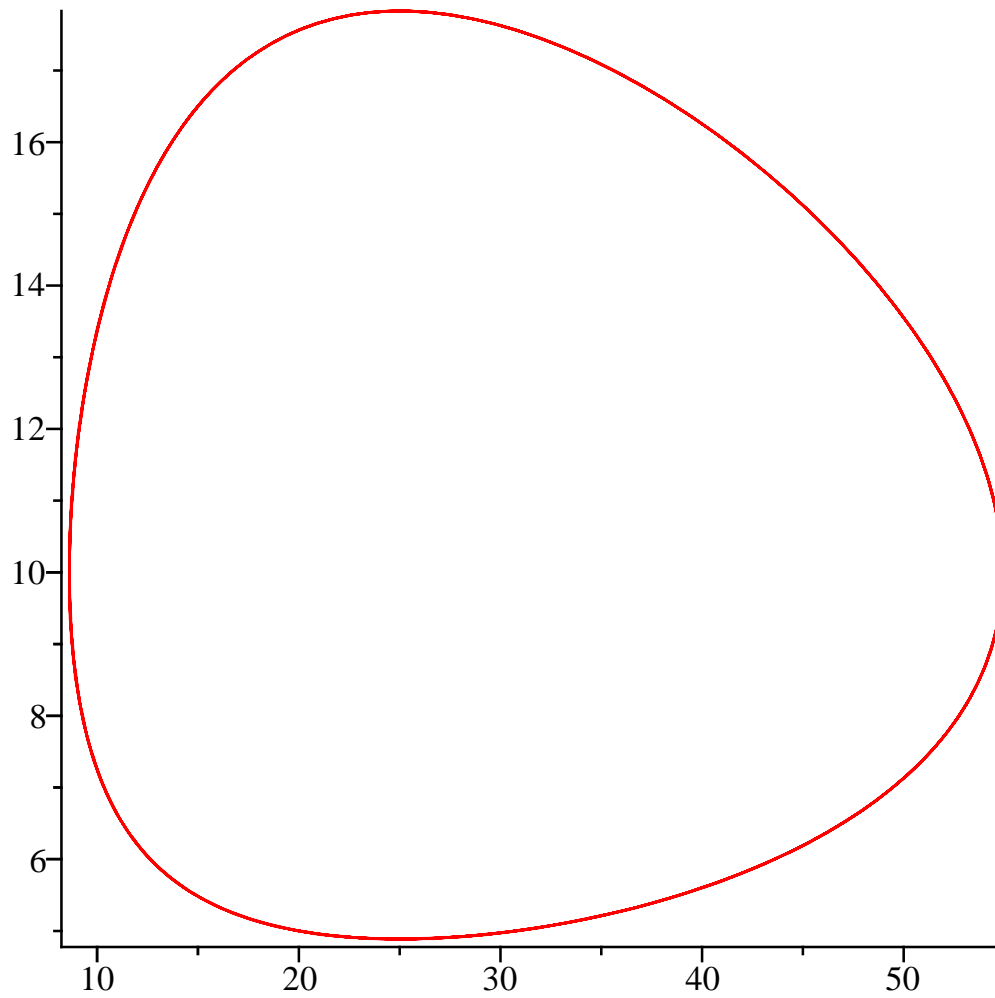
```
> c1 := Vector([20,5]):
```

```
> t, z := RungeKutta(f,a,b,c1,m);
```

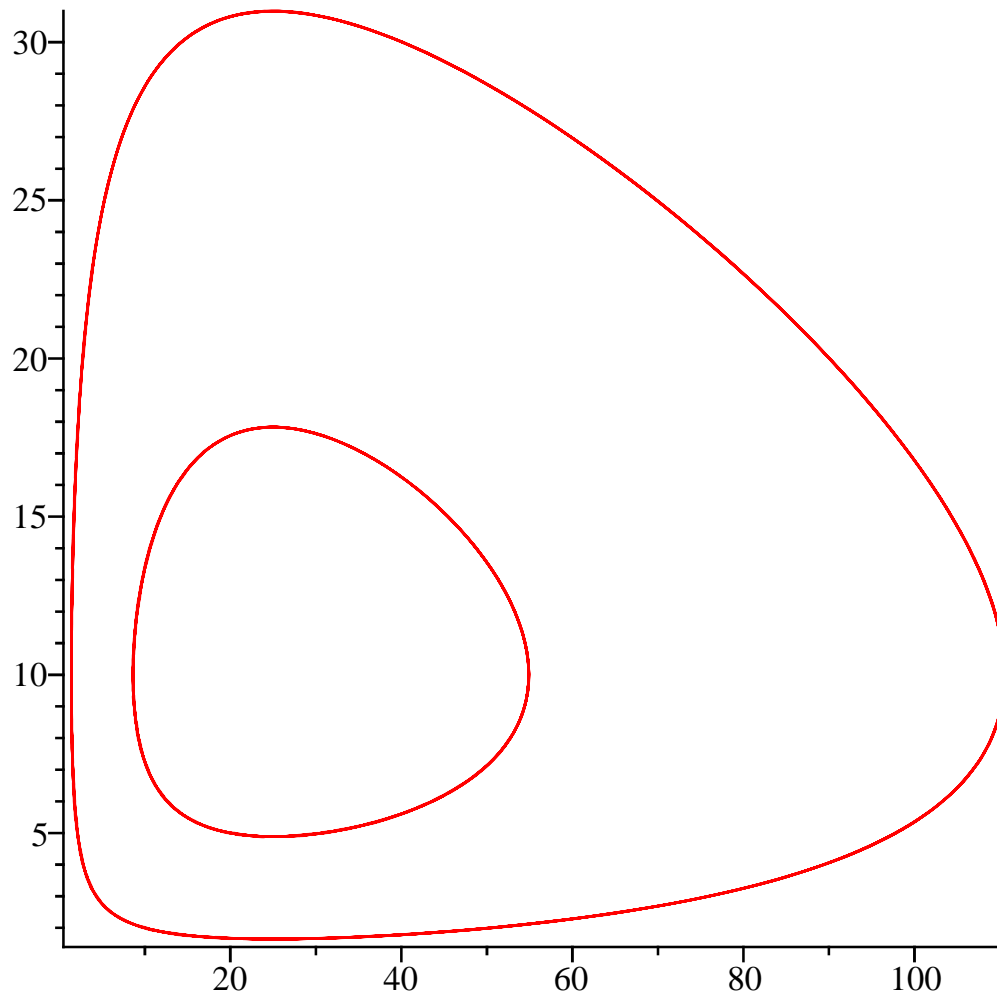
$$t, z := \begin{bmatrix} 1 \dots 1001 \text{ Vector}_{column} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran_order} \end{bmatrix}, x$$

(4)

```
> plot( [seq([z[1][k],z[2][k]],k=1..m)] );
```



```
> p1 := plot( [seq([y[1][k],y[2][k]],k=1..m)] ):  
p2 := plot( [seq([z[1][k],z[2][k]],k=1..m)] ):  
plots[display]({p1,p2});
```



```
> # To find the fixed point  
solve( {f[1](s,u,v)=0, f[2](s,u,v)=0}, {u,v});  
      {u=0.,v=0.}, {u=25.,v=10.}  
>
```

(5)

Project:

Project 5.8, part 1
Project 5.9, part 1