Learning from mistakes

Note Title 2/26/2009

1. $u, v \in \mathbb{R}^n$ are li. indep, A is nonsingular, then Au, Av are li. indep.

Wrong proof. Let $\alpha(Au) + \beta(Av) = 0$

Then $A[\alpha u + \beta v] = 0$

$\because$ u, v are li. indep.,

$\therefore$ $\alpha = \beta = 0$ QED.

Gap

$A(\alpha u + \beta v) = 0$
$\Rightarrow \alpha u + \beta v = 0$
needs proof.

2. (u, v, w) is li. dep, $x \in \langle u, v, w \rangle$, then $\exists$ infinitely many choices of α, β, γ s.t. $x = \alpha u + \beta v + \gamma w$

Wrong proof. Let α', β', γ' be other choice ← exist?

Then $x = \alpha u + \beta v + \gamma w = \alpha' u + \beta' v + \gamma' w$

$\therefore$ $0 = (\alpha - \alpha')u + (\beta - \beta')v + (\gamma - \gamma')w$

$\because$ (u, v, w) is li. dep.

$\therefore$ $\alpha - \alpha'$, $\beta - \beta'$, $\gamma - \gamma'$ are not all zero

could be

$\therefore$ let $k = \alpha - \alpha'$, $l = \beta - \beta'$, $n = \gamma - \gamma'$

... ...

3. let (u, v, w) be a basis for V and $x \in V$.

If $x \notin \langle u, v \rangle$, then (x, u, v) also form a basis for V

Wrong proof. Assume (x, u, v) are li. dep.

$\therefore$ $\exists$ α, β, γ s.t. $\alpha x + \beta u + \gamma v = 0$

and α, β, γ are not all zero

α could be 0

Thus $$x = -\frac{\beta}{\alpha}u - \frac{\gamma}{\alpha}v$$

contradicting $x \notin \langle u, v \rangle$

$\therefore$ (x, u, v) are li. indep

... , . ,

4. $u, v \in \mathbb{R}^n$ are li. indep, A is nonsingular, then Au, Av are li. indep.

Wrong proof. Let $\alpha(Au) + \beta(Av) = 0$

Then $(\alpha A)u + (\beta A)v = 0$

$\because$ u, v are li. indep. $\therefore \alpha A = \beta A = 0$ Wrong step.

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5. W is a subspace of V, $\dim(W) < \dim(V)$

Then $\exists x \in V$ but $x \notin W$.

Wrong proof. Let $\dim(W) = m$, $\dim(V) = m+1$.

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6. Giving an alternative proof.

Simply don't. Concentrate on making one correct proof. If you have two proofs, present the more elegant one.

7. Let (u, v, w) be a basis for V and $x \in V$.

If $x \notin \langle u, v \rangle$, then (x, u, v) also form a basis for V

Wrong proof. Let $\alpha x + \beta u + \gamma v = 0$

but $\alpha x \neq -\beta u - \gamma v$ is given (Not)

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8 W is a subspace of V, $\dim(W) < \dim(V)$

Then $\exists\, x \in V$ but $x \notin W$.

Wrong proof. Let $(w_1, \cdots, w_m)$ be a basis for $W \subset V$

If W span V $\leftarrow$

then $\dim(W) = \dim(V)$

This contradiction proves that

$\exists\, (x_1, \cdots, x_n) = x$ s.t. $x \in V$, $x \notin W$ QED

9. Let u, v, w be li. indep. $x \in \langle u, v, w \rangle$. Then

$\exists$ unique α, β, γ s.t. $x = \alpha u + \beta v + \gamma w$.

Wrong proof. Let $x = \alpha u + \beta v + \gamma w = \alpha' u + \beta' v + \gamma' w$

with $\alpha \neq \alpha'$, $\beta \neq \beta'$ and $\gamma \neq \gamma'$

...

Ω

10 let (u, v, w) be a basis for V and $x \in V$.

If $x \notin \langle u, v \rangle$, then (x, u, v) also form a basis for V

Wrong proof. $\because$ $x \notin \langle u, v \rangle$

$\therefore$ $x \in \langle w \rangle$

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