Eigen Exercises

Prove the following statements.

1. If λ is an eigenvalue of A, then 3λ is an eigenvalue of $3A$.
2. Let λ be an eigenvalue of A. Then $\lambda + \delta$ is an eigenvalue of $A + \delta I$.
3. Let λ be an eigenvalue of A. Then λ^2 is an eigenvalue of A^2.
4. Let λ be an eigenvalue of A that is nonsingular. Then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.
5. Let B be a nonsingular matrix. If λ is an eigenvalue of A, then λ is also an eigenvalue of BAB^{-1}.
6. Let $A, B \in \boldsymbol{R}^{n\times n}$ and A be nonsingular. If λ is an eigenvalue of AB, then λ is also an eigenvalue of BA.
7. Let $\boldsymbol{x}$ be a unit vector. Namely $\boldsymbol{x}^\top \boldsymbol{x} = 1$. If $(A - \lambda I)\boldsymbol{x} = \boldsymbol{b}$, then λ is an eigenvalue of $A - \boldsymbol{b}\boldsymbol{x}^\top$.
8. Let $\boldsymbol{x} \neq \boldsymbol{0}$ be a vector in the null space of A. Then $\boldsymbol{x}$ is an eigenvector of A.
9. If $A^2 = I$ and λ is an eigenvalue of A, then $\lambda = \pm 1$.
10. P is called a projection matrix if $P^2 = P$. Let P be a projection matrix with an eigenvalue λ. Then λ is either 0 or 1.
11. Every nonzero vector is an eigenvector of I.
12. If A is nonsingular then its eigenvalues are nonzero.
13. If 0 is an eigenvalue of $A \in \boldsymbol{R}^{n\times n}$, then $rank(A) < n$.
14. If $A \in \boldsymbol{R}^{n\times n}$ is singular, then 0 is an eigenvalue of A.