

```

> read("d:/343/svd.mpl"); # you can download this program
svd:=proc(A::Matrix)
  local UU, SS, VV, V0, m, n, k, s;
  m, n := LinearAlgebra[Dimension](A);
  UU, s, V0 := LinearAlgebra[SingularValues](A, output = ['U', 'S', 'Vt']);
  k := min(m, n);
  SS := LinearAlgebra[DiagonalMatrix](s[1..k], m, n);
  VV := LinearAlgebra[HermitianTranspose](V0);
  return UU, SS, VV
end proc

```

(1)

```

> UseHardwareFloats:=false:
  Digits:=6:
> A := LinearAlgebra:-RandomMatrix(5,3,generator=-2.0..2.0);
A :=

```

$$\begin{bmatrix}
1.20112187555520045 & 1.85955414079710612 & 0.529436984901638040 \\
-0.0584974051086351032 & 1.83002734173719039 & 1.65350342455607758 \\
1.82866779297178228 & 0.187526076819935384 & -1.49205273482597578 \\
1.88237112704246278 & -0.886007124531806412 & 1.62316774830247690 \\
-1.36954767328980686 & -1.60983838000236190 & 1.25889474557271574
\end{bmatrix}$$

(2)

```

> U, S, V := svd(A);
U, S, V :=

```

$$\begin{bmatrix}
-0.572278 & 0.284080 & 0.0894878 & -0.723793 & 0.244798 \\
-0.285671 & 0.678887 & 0.280711 & 0.588204 & 0.180867 \\
-0.439290 & -0.516951 & -0.257684 & 0.318788 & 0.609730 \\
-0.149704 & 0.326393 & -0.900341 & 0.054017 & -0.239863 \\
0.612800 & 0.290936 & -0.190221 & -0.159994 & 0.691417
\end{bmatrix},$$

$$\begin{bmatrix}
3.64244 & 0 & 0 \\
0 & 3.10936 & 0 \\
0 & 0 & 2.63677 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
-0.712434 & -0.137616 & -0.688115 \\
-0.692713 & 0.294653 & 0.658266 \\
0.112162 & 0.945651 & -0.305250
\end{bmatrix}$$

(3)

```

> # for a rank-deficient matrix:
B := LinearAlgebra:-RandomMatrix(8,4,generator=-1.0..1.0).
      LinearAlgebra:-RandomMatrix(4,6,generator=-1.0..1.0);

```

$$B := \begin{bmatrix} -1.30457 & 1.41704 & -0.620955 & 0.0706224 & -0.145885 & 0.884296 \\ 1.29585 & -1.22180 & 0.361373 & -0.488359 & -0.275295 & -0.724253 \\ -1.05896 & 0.665919 & -0.214108 & 0.0537119 & 0.422682 & 0.700366 \\ -0.424434 & 0.847380 & -0.925776 & -0.307029 & -0.622986 & 0.243715 \\ -0.010826 & -0.077137 & 0.322533 & 0.792977 & 0.499920 & -0.273668 \\ -0.683732 & -0.122068 & 0.241518 & 0.720611 & 1.25171 & 0.102933 \\ -0.004746 & -0.204227 & 0.483146 & 0.450733 & 0.449055 & -0.0684862 \\ -0.271546 & -0.554596 & 1.27935 & 0.940564 & 1.35316 & 0.094012 \end{bmatrix} \quad (4)$$

```
> U, S, V := svd(B):
```

```
> S; # showing "numerical rank" 4
```

$$S = \begin{bmatrix} 3.54492 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.06798 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.682316 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.555846 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.00000413246 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.00000107228 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

```
> # since the 5-th & 6-th singular values are zero
# therefore, the 5-th & 6-th columns of V form a basis for
NullSpace(B)
```

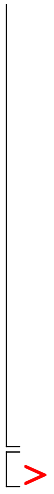
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v5 := V[1..6,5]:
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```
v6 := V[1..6,6]:
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```
v5,v6;
```

$$\begin{bmatrix} 0.694790 \\ 0.253188 \\ -0.217033 \\ 0.0748732 \\ 0.360199 \\ 0.520311 \end{bmatrix}, \begin{bmatrix} 0.104025 \\ 0.581140 \\ 0.256130 \\ -0.448830 \\ 0.363755 \\ -0.502092 \end{bmatrix} \quad (6)$$

```
> B.v5, B.v6; # numerical zero vectors
```



$$\begin{bmatrix} -0.000007 \\ 0.000006 \\ -0.000004 \\ -0.000001 \\ -0.000001 \\ 0.0000042 \\ -0.0000011 \\ 5 \cdot 10^{-7} \end{bmatrix}, \begin{bmatrix} -0.000016 \\ 0.000014 \\ -0.000009 \\ -0.000002 \\ 0. \\ -0.0000018 \\ 4 \cdot 10^{-7} \\ -0.0000027 \end{bmatrix}$$

(7)