

# Solving Least Squares

Note Title

4/9/2009

Given  $Ax = b$   
where  $A$  is  $m \times n$ ,  $m > n$   
(over determined system)

Then: Solve  $A^T A x = A^T b$   
instead (normal equation)

In practical computation, however,  
there is a better way:

~~We can solve~~

1.  $A = QR$  (non-fullspan version)

$$\begin{bmatrix} A \\ m \times n \end{bmatrix} = \begin{bmatrix} Q \\ m \times n \end{bmatrix} \begin{bmatrix} R \\ n \times n \end{bmatrix} \quad Q^T Q = I_{n \times n}$$

$$\begin{aligned} A^T A &= (QR)^T (QR) \\ &= R^T Q^T Q R = R^T R \end{aligned}$$

$$A^T b = (QR)^T b = R^T (Q^T b)$$

$$A^T A x = A^T b$$

$$R^T R x = R^T (Q^T b)$$

$$\boxed{R x = Q^T b}$$

Assume  
 $(R^T)^{-1}$  exist