More examples about Givens transformation

1. Read necessary programs into Maple

```
> read("d:/343/GivensMatrix.mpl");  # read GivensMatrix.mpl into Maple
```

GivensMatrix := **proc**(v::Vector)
    local c, s, r, G;
    r := sqrt(v[1]^2 + v[2]^2);
    if r = 0 **then**
        return `<|>`(`<,>`(1, 0), `<,>`(0, 1))
    else
        c, s := v[1]/r, v[2]/r; **return** `<|>`(`<,>`(c, −s), `<,>`(s, c))
    end if
end proc **(1)**

```
> read("d:/343/ClearMatrix.mpl");
```

ClearMatrix := **proc**(A::Matrix, tol::numeric)
    local m, n, i, j;
    m, n := LinearAlgebra:-Dimension(A);
    for i **to** m **do for** j **to** n **do if** abs(A[i, j]) < tol **then** A[i, j] := 0 **end if end do end do**;
    return A
end proc **(2)**

```
>
> UseHardwareFloats:=false:
  Digits := 10:
```

2. Construct a testing matrix

```
> A := LinearAlgebra:-RandomMatrix(5,5,generator=-9..9,
  outputoptions=[shape=triangular])[1..5,1..5]:
  A[3..5,2] := LinearAlgebra:-RandomVector(3,generator=-9..9):
  A := 1.0*A;
```

$$A := \begin{bmatrix} 8. & -6. & 7. & 8. & 0. \\ 0. & -5. & -9. & 3. & -8. \\ 0. & 2. & -2. & -8. & -5. \\ 0. & 1. & 0. & -9. & -7. \\ 0. & 7. & 0. & 0. & -8. \end{bmatrix} \quad (3)$$

```
> G1 := LinearAlgebra:-IdentityMatrix(5)[1..-1,1..-1]:  # shift-enter
```

```
T := GivensMatrix(A[[4..5],2]):
G1[4..5,4..5] := T:
G1;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.1414213562 & 0.9899494936 \\ 0 & 0 & 0 & -0.9899494936 & 0.1414213562 \end{bmatrix} \tag{4}$$

```
> tol := 0.001:
  A1 := ClearMatrix(G1.A,tol);
```

$$A1 := \begin{bmatrix} 8. & -6. & 7. & 8. & 0. \\ 0. & -5. & -9. & 3. & -8. \\ 0. & 2. & -2. & -8. & -5. \\ 0. & 7.071067811 & 0. & -1.272792206 & -8.909545442 \\ 0. & 0. & 0. & 8.909545442 & 5.798275605 \end{bmatrix} \tag{5}$$

```
> G2 := LinearAlgebra:-IdentityMatrix(5)[1..-1,1..-1]:  # shift-
  enter
  T := GivensMatrix(A1[[3..4],2]):
  G2[3..4,3..4] := T:
  G2;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.2721655270 & 0.9622504486 & 0 \\ 0 & 0 & -0.9622504486 & 0.2721655270 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

```
> A2 := ClearMatrix(G2.G1.A,tol);
```

$$A2 := \begin{bmatrix} 8. & -6. & 7. & 8. & 0. \\ 0. & -5. & -9. & 3. & -8. \\ 0. & 7.348469227 & -0.5443310540 & -3.402069087 & -9.934041733 \\ 0. & 0. & 1.924500897 & 7.351593428 & 2.386381112 \\ 0. & 0. & 0. & 8.909545442 & 5.798275605 \end{bmatrix} \tag{7}$$

```
> G3 := LinearAlgebra:-IdentityMatrix(5)[1..-1,1..-1]:  # shift-
  enter
  T := GivensMatrix(A2[[2..3],2]):
  G3[2..3,2..3] := T:
  G3;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -0.5625439505 & 0.8267673819 & 0 & 0 \\ 0 & -0.8267673819 & -0.5625439505 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

```
> A3 := ClearMatrix(G3.G2.G1.A,tol);
```

$$A3 := \begin{bmatrix} 8. & -6. & 7. & 8. & 0. \\ 0. & 8.888194416 & 4.612860394 & -4.500351605 & -3.712790072 \\ 0. & 0. & 7.747116578 & -0.5664887619 & 12.20247414 \\ 0. & 0. & 1.924500897 & 7.351593428 & 2.386381112 \\ 0. & 0. & 0. & 8.909545442 & 5.798275605 \end{bmatrix} \quad (9)$$

```
> G4 := LinearAlgebra:-IdentityMatrix(5)[1..-1,1..-1]:  # shift-
  enter
  T := GivensMatrix(A3[[3..4],3]):
  G4[3..4,3..4] := T:
  G4;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.9705033332 & 0.2410877023 & 0 \\ 0 & 0 & -0.2410877023 & 0.9705033332 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

```
> A4 := ClearMatrix(G4.G3.G2.G1.A,tol);
```

$$A4 := \begin{bmatrix} 8. & -6. & 7. & 8. & 0. \\ 0. & 8.888194416 & 4.612860394 & -4.500351605 & -3.712790072 \\ 0. & 0. & 7.982575961 & 1.222599536 & 12.41786896 \\ 0. & 0. & 0. & 7.271319399 & -0.625875628 \\ 0. & 0. & 0. & 8.909545442 & 5.798275605 \end{bmatrix} \quad (11)$$

```
> G5 := LinearAlgebra:-IdentityMatrix(5)[1..-1,1..-1]:  # shift-
  enter
  T := GivensMatrix(A4[[4..5],4]):
  G5[4..5,4..5] := T:
  G5;
```

(12)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.6322836571 & 0.7747369723 \\ 0 & 0 & 0 & -0.7747369723 & 0.6322836571 \end{bmatrix} \quad (12)$$

```
> R := ClearMatrix(G5.G4.G3.G2.G1.A,tol);
```

$$R := \begin{bmatrix} 8. & -6. & 7. & 8. & 0. \\ 0. & 8.888194416 & 4.612860394 & -4.500351605 & -3.712790072 \\ 0. & 0. & 7.982575961 & 1.222599536 & 12.41786896 \\ 0. & 0. & 0. & 11.50009068 & 4.096407557 \\ 0. & 0. & 0. & 0. & 4.151043893 \end{bmatrix} \quad (13)$$

```
> Q := LinearAlgebra:-Transpose(G5.G4.G3.G2.G1);
```

$$Q := \begin{bmatrix} 1. & 0. & 0. & 0. & 0. \\ 0. & -0.5625439505 & -0.8023804999 & 0.1260289589 & -0.1544232449 \\ 0. & 0.2250175802 & -0.3805757312 & -0.5671303152 & 0.6949046024 \\ 0. & 0.1125087901 & -0.06501502072 & -0.7316623649 & -0.6691673948 \\ 0. & 0.7875615305 & -0.4551051451 & 0.3565811128 & -0.2132511478 \end{bmatrix} \quad (14)$$

```
> A-Q.R;
```

$$\begin{bmatrix} 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & -1.\ 10^{-9} & 0. & -2.\ 10^{-9} \\ 0. & 0. & -1.\ 10^{-9} & -8.\ 10^{-9} & 0. \\ 0. & 1.\ 10^{-10} & -3.\ 10^{-10} & -9.\ 10^{-9} & -4.\ 10^{-9} \\ 0. & 2.\ 10^{-9} & -1.\ 10^{-9} & 5.\ 10^{-9} & -3.\ 10^{-9} \end{bmatrix} \quad (15)$$

```
>
```

Application of QR decomposition, Solve the homogeneous equation A.x = 0 for $x \neq 0$

1. $A = QR$

2. get a random vector x

3. Solve $R^T y = x$ (forward subs)

   Solve $R z = y$ (back subs)

4. $x = z / \|z\|_2$

```
> A := LinearAlgebra:-RandomMatrix(5,2,generator=-5..5).
       LinearAlgebra:-RandomMatrix(2,4,generator=-5..5);
```

$$A := \begin{bmatrix} -6 & -33 & -4 & 5 \\ 4 & -8 & 16 & 20 \\ -10 & -16 & -24 & -22 \\ 0 & 9 & -4 & -7 \\ -1 & 20 & -12 & -19 \end{bmatrix} \qquad \textbf{(16)}$$

```
> LinearAlgebra:-NullSpace(A);  # Maple calculates a basis for
  null space.
```

$$\left\{ \begin{bmatrix} -\frac{28}{9} \\ \frac{4}{9} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{31}{9} \\ \frac{7}{9} \\ 0 \\ 1 \end{bmatrix} \right\} \qquad \textbf{(17)}$$

```
> # Apply the method
  A := 1.0*A:
  (Q,R) := LinearAlgebra:-QRDecomposition(A):  # step 1
  x := LinearAlgebra:-RandomVector(4,generator=-1.0..1.0);  # step
  2
```

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$$x := \begin{bmatrix} 0.09401778457268994420 \\ -0.09815258713811014780 \\ 0.29549192627261344800 \\ 0.46344477131734063100 \end{bmatrix} \quad (18)$$

```
> y := LinearAlgebra:-ForwardSubstitute(LinearAlgebra:-Transpose
  (R),x);
```

$$y := \begin{bmatrix} 0.007600887386 \\ -0.008005458853 \\ 4.927259171\ 10^{6} \\ -1.427419454\ 10^{7} \end{bmatrix} \quad (19)$$

```
> z := LinearAlgebra:-BackwardSubstitute(R,y);
```

$$z := \begin{bmatrix} -6.930690388\ 10^{15} \\ 1.830870223\ 10^{15} \\ -1.030268029\ 10^{15} \\ 2.942700590\ 10^{15} \end{bmatrix} \quad (20)$$

```
> x := z/LinearAlgebra:-Norm(z,2);
```

$$x := \begin{bmatrix} -0.8866031321 \\ 0.2342126373 \\ -0.1317962296 \\ 0.3764426650 \end{bmatrix} \quad (21)$$

```
> A.x;
```

$$\begin{bmatrix} 5.\ 10^{-9} \\ 0. \\ 4.\ 10^{-9} \\ -1.\ 10^{-9} \\ -2.\ 10^{-9} \end{bmatrix} \quad (22)$$

```
>
```

We want to find another one y

s.t. (1) $Ay = 0$

(2) $x^T y = 0$ $\quad$ ($x \perp y$)

$$\begin{bmatrix} x^T \\ A \end{bmatrix} y = 0 \quad \text{or}$$

$$Ax = 0 \iff QRx = 0 \iff Rx = Q^T 0 = 0$$

$$\therefore \quad \begin{bmatrix} x^T \\ R \end{bmatrix} y = 0 \qquad \begin{bmatrix} x^T \\ R \end{bmatrix} = \begin{bmatrix} \overline{} \\ \diagdown \end{bmatrix}$$

```
> B := <LinearAlgebra:-Transpose(x),R>;
```

$$B := \begin{bmatrix} -0.8866031321 & 0.2342126373 & -0.1317962296 & 0.3764426650 \\ 12.36931687 & 24.73863374 & 27.48737084 & 23.36426522 \\ 0. & 35.74912587 & -15.88850038 & -27.80487566 \\ 0. & 0. & -8.246211248\ 10^{-9} & -1.212678126\ 10^{-9} \\ 0. & 0. & 0. & -4.850712501\ 10^{-9} \end{bmatrix} \quad \textbf{(23)}$$

```
> (Q,R) := LinearAlgebra:-QRDecomposition(B);
```

$$Q, R := \begin{bmatrix} -0.071494193 & -0.05577850850 & 0.9958801836 & -2.700817685\ 10^{-9} \\ 0.9974410162 & -0.00399807043 & 0.0713823160 & -1.93588169\ 10^{-10} \\ 0. & -0.9984351631 & -0.0559216096 & 1.51658879\ 10^{-10} \\ 0. & 0. & -3.032398282\ 10^{-9} & -0.894338519 \\ 0. & 0. & 0. & 0.4473908940 \end{bmatrix},$$
$$\begin{bmatrix} 12.40105096 & 24.65858313 & 27.42645377 & 23.27756298 \\ 0. & -35.80515512 & 15.76109242 & 27.64695618 \\ 0. & 0. & 2.719369450 & 3.597580560 \\ 0. & 0. & 0. & -1.084222447\ 10^{-8} \end{bmatrix} \quad \textbf{(24)}$$

```
> u := LinearAlgebra:-RandomVector(4,generator=-1.0..1.0):
```

```
y := LinearAlgebra:-ForwardSubstitute(LinearAlgebra:-Transpose
(R),u):
z := LinearAlgebra:-BackwardSubstitute(R,y):
u := z/LinearAlgebra:-Norm(z,2);
```

$$u := \begin{bmatrix} -0.3731683017 \\ -0.1054945259 \\ 0.7353135513 \\ -0.5558149912 \end{bmatrix} \quad (25)$$

```
> A.u;
```

$$\begin{bmatrix} 4.\ 10^{-9} \\ 0. \\ 1.\ 10^{-8} \\ 0. \\ 0. \end{bmatrix} \quad (26)$$

```
> # Try to see if we can find the third solution
C := <LinearAlgebra:-Transpose(u),R>;
```

$$C := \begin{bmatrix} -0.3731683017 & -0.1054945259 & 0.7353135513 & -0.5558149912 \\ 12.40105096 & 24.65858313 & 27.42645377 & 23.27756298 \\ 0. & -35.80515512 & 15.76109242 & 27.64695618 \\ 0. & 0. & 2.719369450 & 3.597580560 \\ 0. & 0. & 0. & -1.084222447\ 10^{-8} \end{bmatrix} \quad (27)$$

```
> (Q,R) := LinearAlgebra:-QRDecomposition(C):
v := LinearAlgebra:-RandomVector(4,generator=-1.0..1.0):
y := LinearAlgebra:-ForwardSubstitute(LinearAlgebra:-Transpose
(R),v):
z := LinearAlgebra:-BackwardSubstitute(R,y):
v := z/LinearAlgebra:-Norm(z,2);
```

$$v := \begin{bmatrix} 0.9248286811 \\ -0.2196513377 \\ 0.04279783665 \\ -0.3075931480 \end{bmatrix} \quad (28)$$

```
> A.v;
```

(29)

$$\begin{bmatrix} -0.009635030 \\ -0.010572148 \\ 0.006035768 \\ 0.005098650 \\ 0.012840337 \end{bmatrix} \quad (29)$$

>

The solutions to A.x = 0 is a subspace (null space of A) with basis vectors

> x, u;

$$\begin{bmatrix} -0.8866031321 \\ 0.2342126373 \\ -0.1317962296 \\ 0.3764426650 \end{bmatrix}, \begin{bmatrix} -0.3731683017 \\ -0.1054945259 \\ 0.7353135513 \\ -0.5558149912 \end{bmatrix} \quad (30)$$

>

Anouncement: Thursday, April 16, no class.

Homework:

Solve A.x = 0 for the following matrix A

```
> A := LinearAlgebra:-RandomMatrix(6,3,generator=0..5).
  LinearAlgebra:-RandomMatrix(3,5,generator=0..5);
```

$$A := \begin{bmatrix} 18 & 14 & 29 & 28 & 16 \\ 19 & 13 & 30 & 19 & 21 \\ 16 & 6 & 14 & 10 & 10 \\ 11 & 10 & 23 & 14 & 16 \\ 18 & 15 & 33 & 25 & 21 \\ 37 & 22 & 48 & 40 & 30 \end{bmatrix} \quad (31)$$

>