Note Title 3/12/2009

Wrong proofs

#2. Exam 1 $\because$ $w \in \langle u, v \rangle$

$\therefore$ $\exists\, \alpha, \beta$, not both zero such that

$w = \alpha u + \beta v$ - - - Wrong

#5.

Wrong proof 1. Let $\alpha(3u) + \beta(4v) + \gamma(5w) = 0$

- - - - -

Should start $\alpha u + \beta v + \gamma w = 0$

Wrong proof 2. $\because$ $(3u, 4v, 5w)$ is li. indep.

$\therefore$ $\exists\, \alpha = \beta = \gamma = 0$ s.t. $\alpha(3u) + \beta(4v) + \gamma(5w) = 0$

. . . — Wrong statement.

#9. Let $\alpha u + \beta v = 0$ — Wrong start

Should start with $\alpha L(u) + \beta L(v) = 0$

Matrix representation of L.T.

Let $L : V \to W$ be linear

bases $\mathcal{X}$ $\mathcal{Y}$

$$\begin{tikzcd} v \arrow[r, "L"] \arrow[d, "R_{\mathcal{X}}"'] & w = L(v) \arrow[d, "R_{\mathcal{Y}}"] \\ x \arrow[r] & y = Ax \end{tikzcd}$$

representation of L

How to find $A = R_{\mathcal{Y}\mathcal{X}}(L)$?

$$L\mathcal{X} = \mathcal{Y} \cdot A \qquad \text{— key}$$

$$L(v_1, v_2, \cdots, v_n) = (w_1, w_2, \cdots, w_m)\,[a_1, a_2, \cdots a_n]$$

$$\left.\begin{array}{l} L(v_1) = (w_1, \cdots, w_m)\, a_1 \\ L(v_2) = (w_1, \cdots, w_m)\, a_2 \\ \cdots \end{array}\right\} \text{Find columns } a_1, a_2, \cdots a_n$$

Example p229 #1 / T_3 using basis in (b)

Find representation

$$T_3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}; \quad \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$T_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$$

$$T_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \quad \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$$

The matrix
$$\begin{bmatrix} 1 & 3/2 & 3/2 \\ -1 & -1/2 & -1/2 \end{bmatrix}$$

Example $L : \mathbb{R}_1[z] \to \mathbb{R}_1[z]$

bases $(z+1, z-1)$ $(z+1, z-1)$

defined by
$$L(z+1) = z-1$$
$$L(z-1) = 2z+1$$

(1) Find the matrix
(2) Find $L(2z+3)$ use the matrix
(c) Find general $L(\alpha z + \beta)$

(1) $L(\tau+1) = \tau-1 = (\tau+1, \tau-1)\begin{bmatrix}\alpha\\ \beta\end{bmatrix}, \quad \begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix}$

(2) $L(\tau-1) = 2\tau+1 = (\tau+1, \tau-1)\begin{bmatrix}\alpha\\ \beta\end{bmatrix}, \quad \begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}3/2\\ 1/2\end{bmatrix}$

For (2) : $\begin{cases}\alpha+\beta=2\\ \alpha-\beta=1\end{cases}$ $\quad 2\alpha=3, \quad \alpha=3/2$

$\beta=\frac{1}{2}$

$\therefore$ The matrix is

$$\begin{bmatrix}0 & 3/2\\ 1 & 1/2\end{bmatrix}$$

$$2\tau+3 \xrightarrow{L} ? = -\frac{3}{4}(\tau+1) + \frac{9}{4}(\tau-1)$$

$$\begin{bmatrix}\frac{5}{2}\\ -\frac{1}{2}\end{bmatrix} \longrightarrow \begin{bmatrix}0 & 3/2\\ 1 & 1/2\end{bmatrix}\begin{bmatrix}5/2\\ -1/2\end{bmatrix} = \begin{bmatrix}-3/4\\ 9/4\end{bmatrix}$$

$$\alpha\tau+\beta \xrightarrow{L} ? \quad \frac{3\alpha-3\beta}{4}(\tau+1) + \frac{3\alpha+\beta}{4}(\tau-1)$$

$$\begin{bmatrix}\frac{\alpha+\beta}{2}\\ \frac{\alpha-\beta}{2}\end{bmatrix} \longrightarrow \begin{bmatrix}0 & \frac{3}{2}\\ 1 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}\frac{\alpha+\beta}{2}\\ \frac{\alpha-\beta}{2}\end{bmatrix} = \begin{bmatrix}\frac{3\alpha-3\beta}{4}\\ \frac{3\alpha+\beta}{4}\end{bmatrix}$$

Example P231, #4(b)

$L : V = \left\langle \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \end{bmatrix} \right\rangle \to \left\langle \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\rangle$

If the matrix $B = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -1 & -2 \end{bmatrix}$

What is the L.T.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow{L} ? \qquad \begin{bmatrix} 5x_1 - 3x_2 \\ x_1 \\ -4x_1 + 3x_2 \end{bmatrix}$$

$$\begin{bmatrix} 2x_1 - x_2 \\ x_1 - x_2 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2x_1 - x_2 \\ x_1 - x_2 \end{bmatrix} = \begin{bmatrix} 5x_1 - 3x_2 \\ x_1 \\ -4x_1 + 3x_2 \end{bmatrix}$$

$\therefore \quad L \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

HW P229 #1(b) for T_2

#3

4(c)