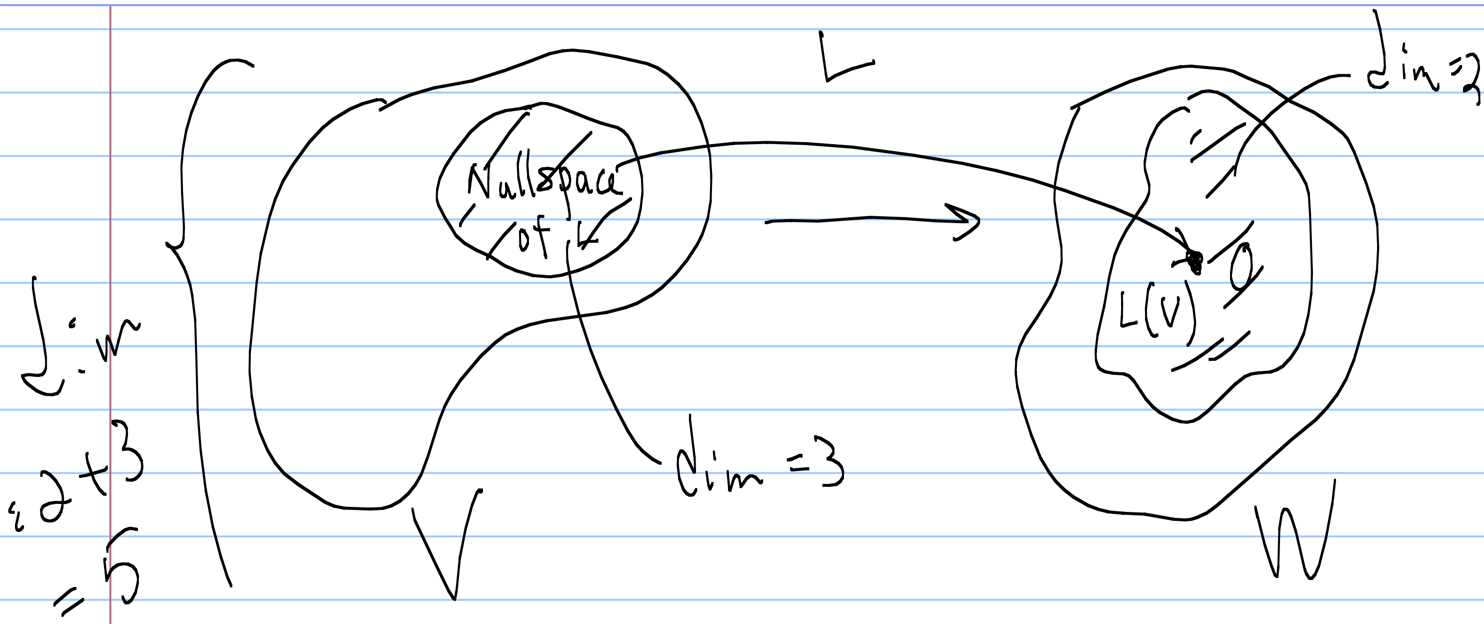
Note Title 2/26/2009



Thm $\dim(\text{Nullspace}(L)) + \dim(\text{Range}(L)) = \dim(V)$

Statement If $L : V \to W$ is linear,

$\dim(\text{NullSpace}(L)) = 2$

$\dim(\text{Range}(L)) = 3$

Then $\dim(V) = 5$

Proof. $\because$ $\dim(\text{NullSpace}(L)) = 2$

$\therefore$ $v_1, v_2 \in V$, li. indep. and span NullSpace(L)

$\therefore$ $\exists\, u_1, \dots, u_m$ s.t. $(v_1, v_2, u_1, \dots, u_m)$ for a basis for V

$\therefore \forall x \in V, \exists \alpha_1, \alpha_2, \beta_1, \dots, \beta_m$ s.t.

$$x = \alpha_1 v_1 + \alpha_2 v_2 + \beta_1 u_1 + \cdots + \beta_m u_m$$

$$\therefore \quad L(x) = \alpha_1 L(v_1) + \alpha_2 L(v_2) + \beta_1 L(u_1) + \cdots + \beta_m L(u_m)$$

$$= 0 + 0 + \beta_1 L(u_1) + \cdots + \beta_m L(u_m)$$

$\therefore$ Range$(L) = \langle L(u_1), \dots, L(u_m) \rangle$

We'll try to prove $(L(u_1), \dots, L(u_m))$ is li. indep

Let $\beta_1 L(u_1) + \cdots + \beta L(u_m) = 0$

$\therefore L(\beta_1 u_1 + \cdots + \beta_m u_m) = 0$

$\therefore \beta_1 u_1 + \cdots + \beta_m u_m \in$ NullSpace(L)

$\therefore \beta_1 u_1 + \cdots + \beta_m u_m = \gamma_1 v_1 + \gamma_2 v_2$

$\therefore \beta_1 u_1 + \cdots + \beta_m u_m - \gamma_1 v_1 - \gamma_2 v_2 = 0$

$\therefore \beta_1 = \beta_2 = \cdots = \beta_m = \gamma_1 = \gamma_2 = 0$

$\therefore (L(u_1), \dots, L(u_m))$ form a basis of

Range(L) $\quad \therefore m = 3$

$\therefore \dim(V) = 2 + 3 = 5$

$\therefore$

QED