Note Title 2/17/2009

$V = \langle v_1, \cdots, v_n \rangle$ (V is spanned by $v_1, \cdots, v_n$)

does imply $(v_1, \cdots, v_n)$ is a basis

Let $(x_1, \cdots, x_n)$ be a basis of V

$\forall v \in V$, $\exists\ \alpha_1, \cdots, \alpha_n$ s.t.

$$v = \alpha_1 x_1 + \cdots + \alpha_n x_n$$

We say $\begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix}$ is a representation of v w.r.t. $(x_1, \cdots, x_n)$

Example $V = \langle 1, t, t^2, t^3 \rangle$

$\forall p \in V$, $\exists$ $p = \alpha_1 (1) + \alpha_2 \cdot t + \alpha_3 t^2 + \alpha_4 t^3$

$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}$ is a vector repres. of p w.r.t. $(1, t, t^2, t^3)$

$p(t) = 2t + 3t^3$: $\begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}$

$$p(t) = 2t + 3t^3 = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}$$

If we use another basis
$(1+t,\ 1-t,\ t+t^2,\ t^3)$

$$p(t) = 2t + 3t^3 = \beta_1(1+t) + \beta_2(1-t) + \beta_3(t+t^2) + \beta_4 t^3$$
$$= (\beta_1+\beta_2) + (\beta_1-\beta_2+\beta_3)t + \beta_3 t^2 + \beta_4 t^3$$

$\beta_1+\beta_2 = 0,\quad \beta_1 - \beta_2 + \beta_3 = 2 \quad \beta_3 = 0,\quad \beta_4 = 3$

$\beta_1 = 1,\ \beta_2 = -1,\ \beta_3 = 0,\ \beta_4 = 3$

$$2t + 3t^2 = [1+t,\ 1-t,\ t+t^2,\ t^3]\begin{bmatrix}1\\-1\\0\\3\end{bmatrix}$$

$\begin{bmatrix}1\\-1\\0\\3\end{bmatrix}$ is the repres. of $2t+3t^3$

w.r.t. $(1+t,\ 1-t,\ t+t^2,\ t^3)$

Every vector $v = [x_1, \ldots, x_n]\begin{bmatrix}\alpha_1\\ \vdots \\ \alpha_n\end{bmatrix}$

basis

representation

Consider the two basis $(1, t, t^2, t^3)$
$(1+t,\ 1-t,\ t+t^2,\ t^3)$

$$1+t = [1, t, t^2, t^3]\begin{bmatrix}1\\1\\0\\0\end{bmatrix}. \qquad 1-t = [1, t, t^2, t^3]\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$$

$$t+t^2 = [1, t, t^2, t^3]\begin{bmatrix}0\\1\\1\\0\end{bmatrix} \qquad t^3 = [1, t, t^2, t^3]\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$[1+t,\ 1-t,\ t+t^2,\ t^3] = [1, t, t^2, t^3]\begin{bmatrix}1 & 1 & 0 & 0\\1 & -1 & 1 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

$$\underbrace{y}_{\text{family of vectors}} = \underbrace{x}_{\text{basis}}\ \underbrace{P}_{\text{matrix}}$$

Thm 3.5.2 : y is a basis

$\iff$ P is nonsingular.

If $y = xP$, $z = \underbrace{y}_{xP}Q$

$$z = x(PQ)$$

$$[1+t,\ 1-t,\ t+t^2,\ t^3] = [1, t, t^2, t^3]\begin{bmatrix}1 & 1 & 0 & 0\\1 & -1 & 1 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

$$[1+t,\ 1+t^2,\ 1+t+2t^2,\ t+t^2+t^3]$$

$$= [1+t,\ 1-t,\ t+t^2,\ t^3]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [1, t, t^2, t^3]\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [1, t, t^2, t^3]\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

HW06 P 138

#1 (e) (f) — Extra credit #8

#5 (a) (b)

set up equations