Note Title 2/10/2009

$\mathcal{X} = (x_1, \cdots, x_t)$ spans V $\Longleftrightarrow$

(1) $V = \langle x_1, \cdots, x_t \rangle = \langle \mathcal{X} \rangle$ or

(2) Every vector $y \in V$ can be written as

$$y = \alpha_1 x_1 + \cdots + \alpha_t x_t$$

and $\beta_1 x_1 + \cdots + \beta_t x_t \in V$ $\forall \beta_1, \cdots, \beta_t \in F$.

Example: W is a subspace of V

$\dim(W) < \dim(V)$

Then $\exists x \in V$ but $x \notin W$.

Proof. Let $(x_1, \cdots, x_n)$ be a basis for W

Then $(x_1, \cdots, x_n)$ is a li. indep family

in V. By Prop 3.4.8 We can extend

$(x_1, \cdots, x_n)$ to a basis $(x_1, \cdots, x_n, x_{n+1}, \cdots, x_m)$

of V. $\because \dim(W) < \dim(V)$ $\therefore m > n$ $\therefore x_m \notin W$

(Otherwise $(x_1, \cdots, x_n, x_m)$ is a li. indep family in W

making $\dim(W) \geq n+1$) QED.

Example Au, Av are li. indep, ~~A is nonsingular~~ $\Rightarrow$ u, v are li. indep

Proof. Let $\alpha u + \beta v = 0$

$$\therefore \quad A(\alpha u + \beta v) = 0$$

$$\therefore \quad \alpha(Au) + \beta(Av) = 0$$

Since Au, Av are li. indep.

$\therefore \quad \alpha = \beta = 0$ QED.

Example u, v are li. dep $\Rightarrow$ $u, u+v$ are li. dep

Proof. $\because$ u, v are li. dep.

$\therefore$ $\exists \mu, \eta$, not both zero, s.t.

$$\mu u + \eta v = 0.$$

$$\therefore \quad (\mu - \eta) u + \eta(u+v) = \mu u + \eta v = 0$$

If $\eta \neq 0$, Then $\mu - \eta, \eta$ are not both zero.
then $u, u+v$ are li. dep.
Otherwise ($\eta = 0$), $\mu - \eta = \mu \neq 0$ $\therefore$
either $\mu - \eta$ or μ is not zero. QED

Homework: choose 4 from each set of problems. (not done in class)
Extra credit, for doing more.