Note Title 2/3/2009

Example R^3 is a vector space

$$R^3 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \; x_1, x_2, x_3 \in R \right\}$$

Example polynomials of degree ≤ 3 is a vector space

$$V = \{ a_0 + a_1 x + a_2 x^2 + a_3 x^3 \mid a_0, \cdots, a_3 \in R \}$$

Proof. V is nonempty, at least $0 \in V$

$$(a_0 + a_1 x + a_2 x^2 + a_3 x^3) + (b_0 + b_1 x + b_2 x^2 + b_3 x^3)$$
$$= (a_0 + b_0) + (a_1 + b_1) x + (a_2 + b_2) x^2 + (a_3 + b_3) x^3 \in V$$

$$\alpha (a_0 + a_1 x + a_2 x^2 + a_3 x^3)$$
$$= (\alpha a_0) + (\alpha a_1) x + (\alpha a_2) x^2 + (\alpha a_3) x^3 \in V$$

Similar, We can verify A1 – A4, S1 – S4

$$0 = 0 + 0x + 0x^2 + 0x^3 \quad (A2)$$
$$(a_0 + a_1 x + a_2 x^2 + a_3 x^3) + (-a_0 - a_1 x - a_2 x^2 - a_3 x^3) = 0 \quad (A3)$$

QED

Example Functions on $[0,1]$ form a VS

$$V = \{ f(x) \mid \text{domain of } f \text{ is } [0,1] \}$$

$$(f+g)x = f(x) + g(x)$$

$$(\alpha f)(x) = \alpha \cdot f(x)$$

Example Prove: If V is a VS,

$u, v, w \in V$, and $u+v = u+w$

then $v = w$

Proof. $\because$ $u+v = u+w$

$\therefore$ $-u + (u+v) = -u + (u+w)$

$\therefore$ $(-u+u)+v = (-u+u)+w$ (A1)

$0 + v = 0 + w$ (A3)

$v = w$ (A2) QED

Example: Let V be a V.S. Then $0 \in V$ is unique.

Proof Let u and v be both zero vectors in V.

$u + v = v$ (A2)

$u + v = u$ (A2)

$\therefore$ $u = v$. $\therefore$ 0 is unique QED

HW additional problems

1(d) P99

$\alpha u = 0 \Rightarrow \alpha = 0$ or $u = 0$

#4 P100

Prove (a)

And explain why (b), (d) are not