Note Title 1/29/2009

Prove : $A - A(A+B)^{-1}A = B - B(A+B)^{-1}B$

Proof. $\text{LHS} = A\left[I - (A+B)^{-1}A\right]$

$= A\left[(A+B)^{-1}(A+B) - (A+B)^{-1}A\right]$

$= A(A+B)^{-1}\left[\cancel{A}+B-\cancel{A}\right]$

$= A(A+B)^{-1}B = (A+B-B)(A+B)^{-1}B$

$= (A+B)(A+B)^{-1}B - B(A+B)^{-1}B = \text{RHS}$

QED

Summary on "A is nonsingular" for $n \times n$ matrix A

A is nonsingular

$\Leftrightarrow$ $\text{Rank}(A) = n$

$\Leftrightarrow$ A^{-1} exists

$\Leftrightarrow$ $A_R = I$

$\Leftrightarrow$ $Ax = b$ has a unique solution

$\Leftrightarrow$ $Ax = 0$ has only trivial solution $x = 0$

$\Leftrightarrow$ $A = E_1 E_2 \cdots E_k$ (elementary matrices)

Example : Prove : If AB is singular, then either A or B is singular

Proof $\because$ AB is singular

$\therefore$ $(AB)x = 0$ has a nontrivial solution $x \neq 0$.

$(AB)x = A(Bx) = 0$

If $Bx = 0$ then B is singular because it has a nontrivial solution

If $y = Bx \neq 0$ then $Ay = 0$ has a nontrivial solution $\therefore$ A is singular

Consequently either A or B is singular

QED.

A is singular (A is $n \times n$)

$\Leftrightarrow$ $\text{Rank}(A) < n$

$\Leftrightarrow$ A^{-1} doesn't exist

$\Leftrightarrow$ $A_R \neq I$ (some rows are zeros)

$\Leftrightarrow$ $Ax = b$ is either inconsistent, or has infinitely many solutions

$\Leftrightarrow$ $Ax = 0$ has a nontrivial solution

$\Leftrightarrow$ A can't be written as a product of elementary matrices

Read Chap 2

HW: p87 #5

p92 #2

For p93 prove #5, 6, 8, 10

p95 #7, 9 (make ~~a~~ choices and state your reasons)

Due Thur. next week.