Example: The linear system on page 45

```
> A := <<2, 1, 4>|<3, 2, 6>|<1, 3, 2>>;
```

$$A := \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 4 & 6 & 2 \end{bmatrix} \quad \textbf{(1)}$$

```
> b := <5, 7, 10>;
```

$$b := \begin{bmatrix} 5 \\ 7 \\ 10 \end{bmatrix} \quad \textbf{(2)}$$

```
> with(LinearAlgebra) :
> <A|b>;
```

$$\begin{bmatrix} 2 & 3 & 1 & 5 \\ 1 & 2 & 3 & 7 \\ 4 & 6 & 2 & 10 \end{bmatrix} \quad \textbf{(3)}$$

```
> ReducedRowEchelonForm( <A|b> );    #``rank(<A|b>)=2
```

$$\begin{bmatrix} 1 & 0 & -7 & -11 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(4)}$$

```
>
```

$$\begin{cases} x_1 - 7x_3 = -11 \\ x_2 + 5x_3 = 9 \end{cases}$$

Let $x_3 = \gamma$. Then

$$\begin{cases} x_1 = -11 + 7\gamma \\ x_2 = 9 - 5\gamma \\ x_3 = \gamma \end{cases} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 9 \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 7 \\ -5 \\ 1 \end{bmatrix}$$

Page 73, Problem 1(e) Solve $Ax=0$

> $A := \langle\langle 4, 1, 1\rangle | \langle -12, -1, 10\rangle | \langle 32, -1, 0\rangle | \langle 4, 1, 1\rangle\rangle;$

$$A := \begin{bmatrix} 4 & -12 & 32 & 4 \\ 1 & -1 & -1 & 1 \\ 1 & 10 & 0 & 1 \end{bmatrix} \quad (5)$$

> $R := ReducedRowEchelonForm(A);$

$$R := \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (6)$$

>

(n = 4 > rank(A)=3 , therefore, there are nontrivial solutions to Ax=0)
This implies the system is row-equivalent to (press F5 to enter math, press again to escape)

$$\begin{aligned} x \quad\quad + w &= 0 \\ y \quad\quad &= 0 \\ z \quad &= 0 \end{aligned}$$

The solution is $x = -\gamma,\ \ y = z = 0\ \ w = \gamma$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \gamma \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Page 86, Problem 2(c) Sove $Ax=b$

> $A := \langle\langle 1, 2, 3, 0\rangle | \langle 2, 1, 2, 1\rangle | \langle 2, 1, 2, 1\rangle\rangle;$

$$A := \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix} \quad (7)$$

> $b := Vector([1, 2, 3, 0]);$

(8)

$$b := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix} \quad (8)$$

> *ReducedRowEchelonForm*(⟨*A*|*b*⟩);

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

>

The linear system is equivalent to (x_1 for x_1)

$$\begin{aligned} x_1 \quad\quad &= 1 \\ x_2 + x_3 &= 0 \end{aligned}$$

So the solution is $x_1 = 1, \quad x_2 = -\gamma, \quad x_3 = \gamma.$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is a particular solution to the system, and $\gamma \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ is the general solution to the homogeneous system $Ax = 0$.

In homework, you are not allowed to solve the system by LinearSolve

> *LinearSolve*(*A*, *b*);

$$\begin{bmatrix} 1 \\ -\_t_3 \\ \_t_3 \end{bmatrix} \quad (10)$$

>

>

Computational homework: Follow the above examples, do
p73, 1(f)
p86, 2(b)

Proof problems:
(1) Prove $(I + A)^{-1}$ and A commute.

(2) Extra credit: Prove $A - A(A+B)^{-1}A = B - B(A+B)^{-1}B$