Note Title 1/13/2009

$A \in F_{m,n}$ $\quad F = \begin{cases} \mathbb{R} & \text{real \#s} \\ \mathbb{C} & \text{complex \#s} \end{cases}$

A is a $m \times n$ matrix of entries in F

$A \in F_{m,n}$ $\quad B \in F_{n,k}$

$C = AB \in F_{m,k}$

Example $\quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix}$

$\quad 3 \times \underbrace{2 \qquad\qquad 2}_{\text{match}} \times 2$

AB is 3×2

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 18 & 21 \\ 40 & 47 \\ 69 & 81 \end{bmatrix} \leftarrow AB$$

$$\begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix}$$

$AB = BA$ sometimes

If so we say A and B commute

$AI = IA = A$

$AO = OA = O$

$AA^{-1} = A^{-1}A = I$

$(A+B)^2 \overset{?}{=} A^2 + 2AB + B^2$

Answer: sometimes if A, B commute

X is inverse of A

$\iff XA = I$ or $AX = I$

Thm 1.4.4

$$XA = I \text{ and } AY = I$$

$$\Rightarrow X = Y$$

Proof. $XA = I$

$$XAY = IY = Y$$

$$XAY = X(AY) = XI = X$$

$\therefore X = Y$ QED

1.4.5 $(AB)^{-1} = B^{-1}A^{-1}$

Note: $(AB)^{-1} \neq A^{-1}B^{-1}$ in general

Proof. $(AB)(B^{-1}A^{-1})$

$$= A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$$

$\therefore B^{-1}A^{-1}$ is the $\underbrace{\text{inverse of}}_{B^{-1}A^{-1} = (AB)^{-1}}$ AB

Example: If A is nonsingular and $\alpha \neq 0$

The $(\alpha A)^{-1} = \frac{1}{\alpha} A^{-1}$

Proof $(\alpha A)\left(\frac{1}{\alpha} A^{-1}\right) = \alpha \cdot \frac{1}{\alpha} AA^{-1} = 1 \cdot I = I$

QED.

Example If A, B are nonsingular

Then $B^{-1} = A^{-1} - B^{-1}(B-A)A^{-1}$

Proof $B\left(A^{-1} - B^{-1}(B-A)A^{-1}\right)$

$= B\left(A^{-1} - B^{-1}(BA^{-1} - I)\right)$

$= B(\cancel{A^{-1}} - \cancel{A^{-1}} + B^{-1}) = BB^{-1} = I$

QED

#12, P22

Prove $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$, $n = 1, 2, 3, \dots$

By induction

Review : Induction

step 1. Verify it is true for $n=1$

step 2. Assume it is true for $n=k$
prove it is true for $n=k+1$

Proof. For $n=1$, $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}^1 = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$

Assume $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}^k = \begin{bmatrix}1 & k\\ 0 & 1\end{bmatrix}$ $(n=k)$

$$\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}^{k+1} = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}^k \cdot \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & k\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & k+1\\ 0 & 1\end{bmatrix}$$

QED

Problem 19, p23 is similar

HW01, p22 #13, 19*
p29, #6,

Simplified version of #8:

If $S^3 = 0$ then

$$(I - S)^{-1} = I + S + S^2$$

#11, #16