Note Title 3/3/2009

Exam I Tue. Mar 10.

1. Basic matrix algebra (Chap 1)
2. Linear systems (chap 2)
3. Li. dep. & Li. indep. (chap 3)
4. Vector spaces and bases (chap 3)
5. Lin. Trans. (chap 5.1)

All short proofs

Homework 8 is due Thur. Mar 12

§ 5.2. Representation of L.T.

a "vector" in V $\Longleftrightarrow$ a vector $\begin{bmatrix} * \\ * \\ \vdots \\ * \end{bmatrix} \in F_{n,1}$

a L.T. $\Longleftrightarrow$ matrix

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$T: \mathbb{R}_{21} \to \mathbb{R}_{21}$

$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}3\\7\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}5\\2\end{bmatrix}$

If the bases are same

$\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)$

Every vector in $\mathbb{R}_{21}$ can be written as

$$\Phi\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)\underbrace{\begin{bmatrix}\alpha_1\\\alpha_2\end{bmatrix}}_{\text{rep}}$$

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)\begin{bmatrix}\alpha_1\\\alpha_2\end{bmatrix}$$

$$= T\left(\begin{bmatrix}1\\0\end{bmatrix}\alpha_1 + \begin{bmatrix}0\\1\end{bmatrix}\alpha_2\right)$$

$$= T\begin{bmatrix}1\\0\end{bmatrix}\alpha_1 + T\begin{bmatrix}0\\1\end{bmatrix}\alpha_2 = \begin{bmatrix}3\\7\end{bmatrix}\alpha_1 + \begin{bmatrix}5\\2\end{bmatrix}\alpha_2$$

$$= \begin{bmatrix}3 & 5\\7 & 2\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\end{bmatrix}$$

$\therefore \begin{bmatrix}3 & 5\\7 & 2\end{bmatrix}$ is the rep of T w.r.t. bases.

$T: V \to W$

$$\begin{array}{ccc} \overset{\in W}{u} & = & T(\overset{\in V}{v}) \quad \text{L.T.} \\ \text{rep} \downarrow & & \downarrow \text{rep} \\ y & = & A \; x \\ & & \uparrow \text{rep of } T \end{array}$$

Consider the same L.T. T above

$$T: \mathbb{R}_{2,1} \longrightarrow \mathbb{R}_{2,1}$$

basis $\left(\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right)$ $\qquad$ $\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\0\end{bmatrix}\cdot 1 + \begin{bmatrix}0\\1\end{bmatrix}\cdot 1\right) = \begin{bmatrix}3\\7\end{bmatrix} + \begin{bmatrix}5\\2\end{bmatrix} = \begin{bmatrix}8\\9\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\-1\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\0\end{bmatrix}\cdot 1 + \begin{bmatrix}0\\1\end{bmatrix}(-1)\right) = \begin{bmatrix}3\\7\end{bmatrix} - \begin{bmatrix}5\\2\end{bmatrix} = \begin{bmatrix}-2\\5\end{bmatrix}$$

$$T\left(\left(\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right)\begin{pmatrix}\alpha_1\\\alpha_2\end{pmatrix}\right) = \left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right) A \begin{bmatrix}\alpha_1\\\alpha_2\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right) = \left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right) A$$

$$T\begin{bmatrix}1\\1\end{bmatrix} = \left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)\begin{bmatrix}8\\9\end{bmatrix}, \quad T\begin{bmatrix}1\\-1\end{bmatrix} = \left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)\begin{bmatrix}-2\\5\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right) = \left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right)\begin{bmatrix}8 & -2\\ 9, & 5\end{bmatrix}$$

$$T: R_5(z) \longrightarrow R_4(z) \qquad T(p) = p'$$

basis $\quad (1, z, z^2, z^3, z^4) \qquad (1, z, z^2, z^3)$

$$T(1) = 0 = (1, z, z^2, z^3)\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$T(z) = 1 = (1, z, z^2, z^3)\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$T(z^2) = 2z = (1, z, z^2, z^3)\begin{bmatrix}0\\2\\0\\0\end{bmatrix}$$

$$T(z^3) = 3z^2 = (1, z, z^2, z^3)\begin{bmatrix}0\\0\\3\\0\end{bmatrix}$$

$$T(1, z, z^2, z^3, z^4) = (1, z, z^2, z^3)\begin{bmatrix}0 & 1 & 0 & 0 & 0\\ 0 & 0 & 2 & 0 & 0\\ 0 & 0 & 0 & 3 & 0\\ 0 & 0 & 0 & 0 & 4\end{bmatrix}$$

$$T\,X = Y \cdot A$$

$$R_{yx}T = A$$

Example $\quad T: P_2(z) \to P_3(z)$

basis $\quad (1, z, z^2) \qquad (1, z, z^2, z^3)$

$$T(p) = (z+10)\cdot p$$

$$T(1) = (z+10)\cdot 1 = 10 + z = (1, z, z^2, z^3)\begin{bmatrix} 10 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$T(z) = (z+10)\cdot z = 10z + z^2 = (1, z, z^2, z^3)\begin{bmatrix} 0 \\ 10 \\ 1 \\ 0 \end{bmatrix}$$

$$T(z^2) = (z+10)z^2 = 10z^2 + z^3 = (1, z, z^2, z^3)\begin{bmatrix} 0 \\ 0 \\ 10 \\ 1 \end{bmatrix}$$

$$T(1, z, z^2) = (1, z, z^2, z^3)\begin{bmatrix} 10 & 0 & 0 \\ 1 & 10 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \\ & & 1 \end{bmatrix}$$

Application:

convolution matrix

Given $\quad p(z) = 20 + 32z - 7z^2 - z^3$

Find $\quad q(z) \quad$ s.t. $\quad p(z) = (z+10)\,q(z) = T(q)$

(Namely $\quad q(z) = p(z) \div (z+10)$)

$\therefore \quad T(g) = p$

$$\begin{bmatrix} 10 & 0 & 0 \\ 1 & 10 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 32 \\ -7 \\ -1 \end{bmatrix}$$

$\uparrow$

by maple $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \longrightarrow 2 \cdot 1 + 3\tau + (-1)\tau^2 = g(\tau)$