NOTES

A Study in Step Size

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While experimenting with giving polar plots some "texture," I discovered an interesting effect that might intrigue students: high sensitivity to step size. The base equation is that used to generate the butterfly curve (see **[1]**, for example)

$$r(\theta) = e^{\cos(\theta)} - 2\cos(4\theta),$$

but the technique can be applied, and the same effect observed, using almost any polar equation.

The "texture" is obtained by multiplying this base curve by a rapidly varying sinusoidal factor, in this case by $\sin^4(\lambda\theta)$, where $\lambda = 99999999$. The fourth power was chosen to keep the factor non-negative and small. The value of λ was chosen arbitrarily; any large number would produce the same effect.

Data sets, consisting of points (x_n, y_n) where

$$\rho(\theta) = \left(e^{\cos(\theta)} - 2\cos(4\theta)\right)\sin^4(\lambda\theta)$$

and

$$x_n = \rho(\theta_n)\sin(\theta_n)$$

$$y_n = \rho(\theta_n)\cos(\theta_n),$$

were produced by setting $\theta_0 = 0$, and $\theta_n = \theta_{n-1} + h$ where h denotes the step size, for $0 \leq \theta < 2\pi$. (Reversing the sine and cosine from their "usual" positions rotates the butterfly 90^o into the upright position shown.) These data sets contain roughly 11,500 to 42,000 points.

The plots shown on the cover of this issue were produced with step sizes as follows:

$$h_1 = 0.00015 \quad h_2 = 0.0003$$

$$h_3 = 0.0005 \quad h_4 = 0.00055$$

The plots in the following Figure used these step sizes:

$$h_5 = 0.0007 \quad h_6 = 0.000169$$

$$h_7 = 0.000711 \quad h_8 = 0.00071$$

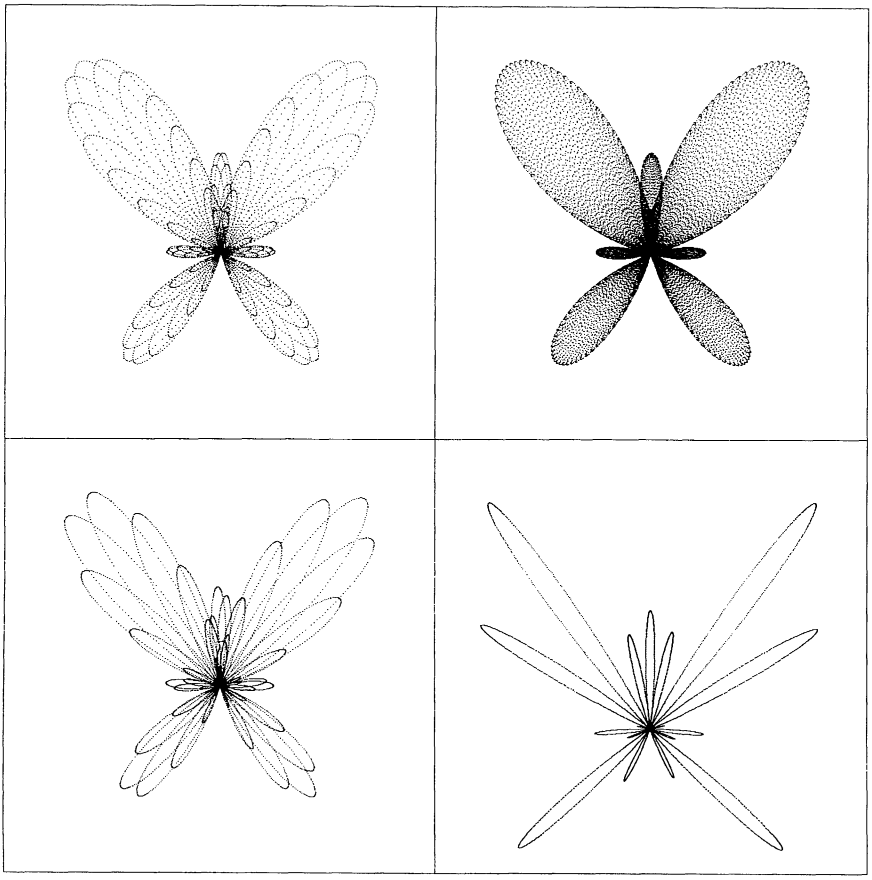

FIGURE
Four butterfly curves

Watching the dynamics of the plotting of the sequentially-generated points (x_n, y_n) is interesting in and of itself. Students might enjoy experimenting with different equations, values of λ, and step sizes.

REFERENCE

1. T. H. Fay, The butterfly curve, *American Mathematical Monthly* 96 (1989), 442–443.