Additional Exercises for Math 322

Prove the following assertions. All lower case letters represent integers.

1. If $a = b + km$ then $a \equiv b \pmod{m}$.
2. If $ab|c$, then $a|c$ or $b|c$.
3. If $ab|cd$ and $(a,c) = 1$, then $a|d$.
4. If a and b are even, then (a,b) is even.
5. If $a^2|b$, then $a|b$.
6. If $7a \equiv b \pmod{5}$, then $2a \equiv b \pmod{5}$.
7. If $a|b$, then $a|b^n$, $\forall n > 1$.
8. If $a|b$, then $(-a)|b$.
9. If $d|a$ and $d|b$, then $d|(a\ mod\ b)$.
10. If $a|b$, then $a|(-b)$.
11. If $d|(a+b)$ and $d|(a-b)$, then $d|2a$ and $d|2b$.
12. If p is prime and $p \not| m$, then $(p,m) = 1$.
13. If p is an odd prime, then $p \not| (x+1, x-1)$, $\forall x$.
14. If $a \equiv b \pmod{m^2}$, then $a \equiv b \pmod{m}$.
15. $n^2 \equiv n \pmod{2}$ for all n.
16. If $a|b$ and $u|v$, then $au|bv$.
17. If $u|a$ and $u|b$ then $u|(a,b)$.
18. If $a \equiv b \pmod{\text{m}}$, $b \equiv c \pmod{\text{m}}$ then $a \equiv c \pmod{\text{m}}$.
19. If a is odd, then (a,b) is odd.
20. If $3 \not| b$, then $(a,b) = (3a,b)$.
21. $c(a,b) = (ac,bc)$, $\forall c > 0$.
22. $(a+bn, b) = (a,b)$, $\forall n \geq 0$.
23. If $d|(a+b)$ and $d|(a-b)$, then $d|(2a, 2b)$.
24. If there exist x and y such that $ax + by = 1$, then a and b are relatively prime.
25. If $ax + by = c$, then $(a,b) < |c|$
26. If $(a, mn) = 1$, then $(a,m) = (a,n) = 1$.
27. If $(a,m) = (a,n) = 1$, then a and mn are relatively prime.
28. If n is odd, then $3 \,|\, (7^n + 5^n)$.
29. If $3 \not| a$, then $3 \,|\, (a^n - a)$ for all odd n.
30. If $(6,a) = 1$, then $a^n \equiv a \pmod{6}$ for all odd n
31. If $p \neq q$ are primes, then $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$.
32. If $3 \not| x$, then $3|\, x^2 - 1$.
33. If n is even and a is a perfect square, then the last digit of a^n is 1.
34. If $(a, 10) = 1$, then $a^3 = a^{-1}$ mod 10.
35. $10|(3^{4k+1} + 7^{4k+1})$, $\forall k > 0$
36. If $(a, 63) = 1$, then $a^6 \equiv 1 \pmod{63}$.
37. If $(a, 24) = 1$, then $a^4 \equiv 1 \pmod{24}$.
38. $7 \not| (r^2 + 1)$, $\forall r$.
39. If $(n, 27) = 1$, then $27 \not| n^6 + 1$.
40. If $(a,b) = 1$, then $(a^2, b^2) = 1$.