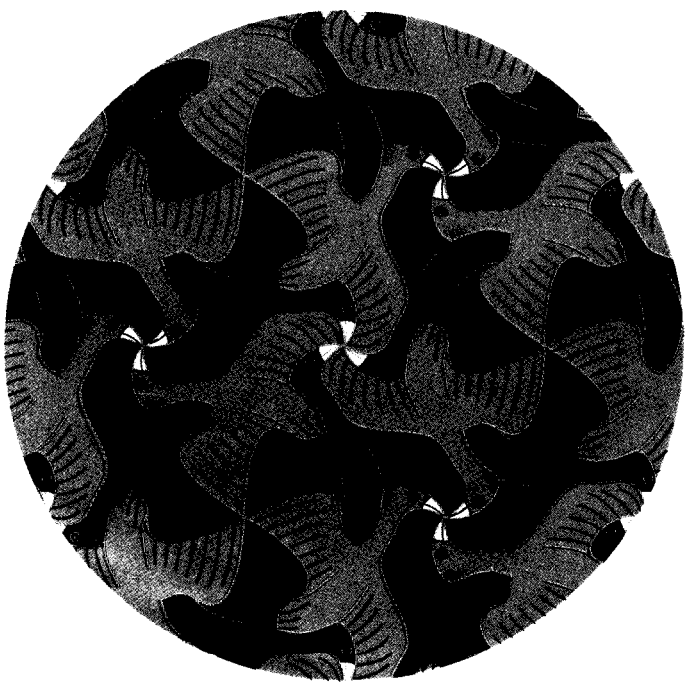
Desktop Science

TEXT AND ILLUSTRATIONS BY OLAV MARTIN KVERN

Escher Sketch. An adventure in the world of tessellations

Before I started school, I spent many a winter afternoon with my father's books. I could read some of them, but the ones I couldn't read at all fascinated me the most.They were his books on mathematics-he taught high school algebra and calculus. As far as I could tell, they were written in an entirely different language from the one Id just learned to read. The illustrations in these textbooks, however, caught my imagination and have not let go to this day.

Look at the covers or chapter dividers in almost any text on mathematics, and you'll see the pictures that caught my eye: the geometric artwork of Dutch artist M. C. Escher. His work is characterized by interlocking figures-usually animals-that cover a plane. These patterns are called "tessellations," from the Latin word "tessera," which means 'cube" and refers to the small pieces of glass, stone, or ceramic material used to make up a mosaic. I find the patterns created by interlocking geometric shapes fascinating, especially when, as in much of Escher's work, the underlying geometric pattern is not immediately obvious.

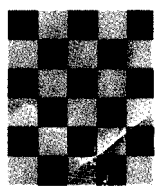


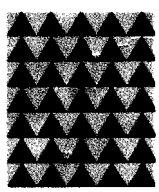
Today's software tools make creating these patterns easy (maybe too easy-I've lost entire days this way). Adobe Illustrator is excellent for working with tessellations, but just about any program with drawing tools will do-even Adobe PageMaker or Adobe Photoshop.

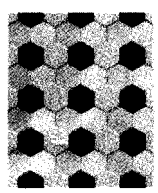
But I'm getting ahead of myself. Before I start talking about the ways that Escher constructed his artwork, I'd better cover the basic rules and techniques for creating tessellations.

Tessellation basics

When a geometric shape evenly covers a plane, it "tessellates" the plane. A checkerboard is a simple example of a single shape, a square, tessellating a plane. Only three regular polygons (that is, polygons having sides of equal lengths) can evenly tessellate a plane by themselves: triangles, squares, and hexagons (at right). When you use one of these three single types of equilateral polygons to divide a plane, it's called a *regular tessellation.*







How about using more than one type of regular polygon? To find out which patterns of regular polygons work together, try arranging them around a single point, as shown in figure 1 on page 45. Using the pat-

Desktop Science

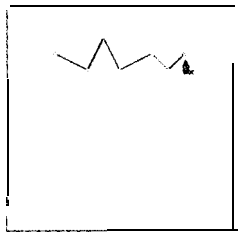
Tessellation tips

Tessellations aren't, for the most part, things you can assemble by eye-even at the highest magnifications.Gaps and misalignments stand out more in geometric art than in other types of illustration. Luckily, in Illustrator, you can work with paths "by the numbers" using the Transform palette and the Scale, Rotate, Reflect, and Shear dialog boxes. Here are a few of my favorite Illustrator tips.

- Turn on Snap To Point (from the View menu in version 8.0, in General Preferences in version 7.0). If you don't, you'll have trouble getting polygon vertices to snap into the right positions.
- Use the Pathfinder palette-especially the Intersect command-to pull complex shapes out of overlapping regular polygons.
- Use grids and, even better, Snap To Grid (choose it from the View menu).
- Use the Align palette to get objects into position. Even if you can't find an edge to align to, you can usually draw temporary paths to aid in aligning objects (see "Impossible alignment:' page 46).
- Set your stroke weight to zero while you're drawing and aligning the polygons in your tessellation (because Illustrator considers the width of the stroke when aligning objects). It's a good idea to work in artwork mode rather than preview mode for this part. Once you've got things arranged the way you want them, you can set the stroke weight the way you want it.
- Create polygons to exact diameters—that way, you'll always know just how far to move them into position.
- You can often create patterns by drawing a bounding box based on the center points of the polygons used in a tessellation.

Creating an Escher-style tessellation

1. Start with a square grid (Illustrator's grid feature can come in handy, as would Smart Guides in 8.0). Draw one side of the tile.
2. Duplicate the side (Alt- or Option-drag) and move it to the opposite side of the square.
3. Rotate the two paths 90" to make them form the remaining sides of the polygon.
4. Join the endpoints of the paths to create a closed path. Duplicate and move the path to create a tiling.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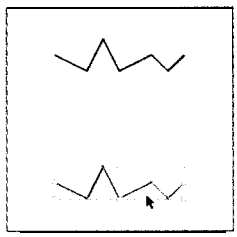
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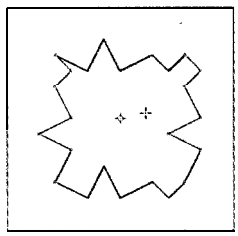
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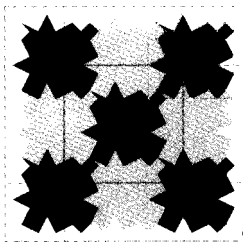
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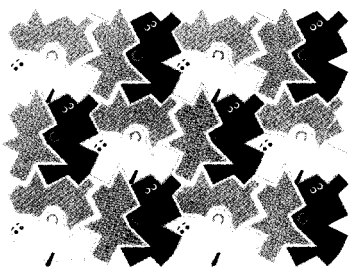
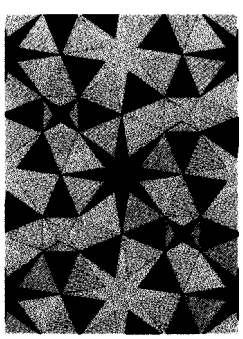
Alternatively, you can embellish the polygon before you duplicate it. Does the shape of the polygon suggest anything to you? (It's a bit like looking at clouds to see if you see a dragon, a boat, or a freight train.)





Tessellations created with star polygons are the basis for Islamic geometric art-in fact, the mosaic tiling of the Alhambra, the 13th-century Moorish palace and fortress in Seville, Spain, was a particular inspiration to Escher.

The geometry of filling space

figure 1

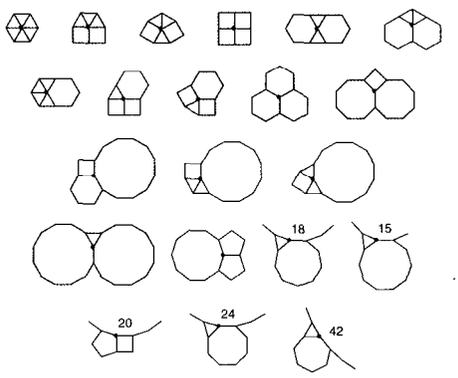
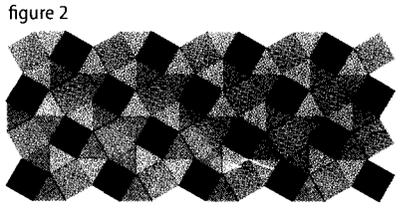


figure 2



There are 21 possible ways (figure 1) to arrange regular polygons around a point. Counting the three regular polygons that tessellate a plane on their own, eleven of these combinations (highlighted in blue here) can be used to create semiregular tessellations, in which every point is surrounded by the same arrangement of polygons (figure 2).

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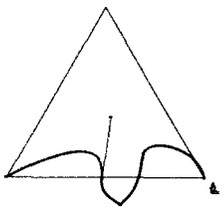

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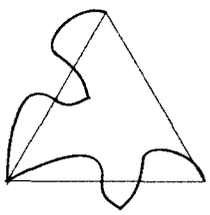

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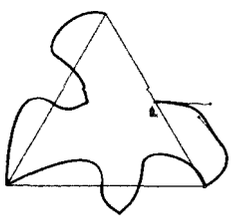

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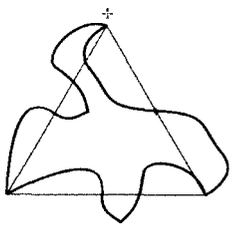

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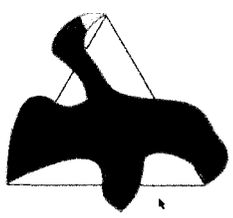
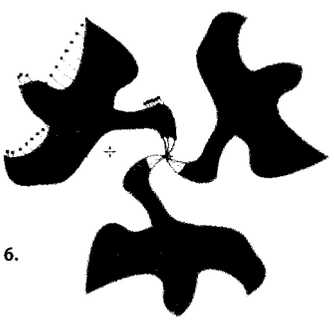
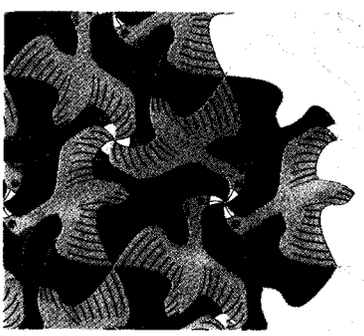


Making birds. A5 **an added twist, Escher sometimes changed** *half* **a side, and then rotated or reflected that change around the midpoint of the side. Here's how to do it.**

1. Draw a path, making sure that the endpoints of the path fall on the corners of the triangle.
2. Duplicate the path, and then rotate it 60° around one of its endpoints. (My favorite method is to use the rotate tool, Alt- or Option-click on the endpoint, type the angle in the text box, and click Copy.)
3. Draw a new path from one corner of the triangle to the midpoint of the remaining side.
4. Duplicate the path, and then rotate the copy 180° around that midpoint. Ioin the endpoints of the paths.
5. Embellish the"bird"shape.
6. Duplicate the bird, and then rotate the duplicate 120° around any of the triangle's corners.

6.


Here's how the birds fit together. A simple tessellation of regular triangles is the basis for this more complex pattern.



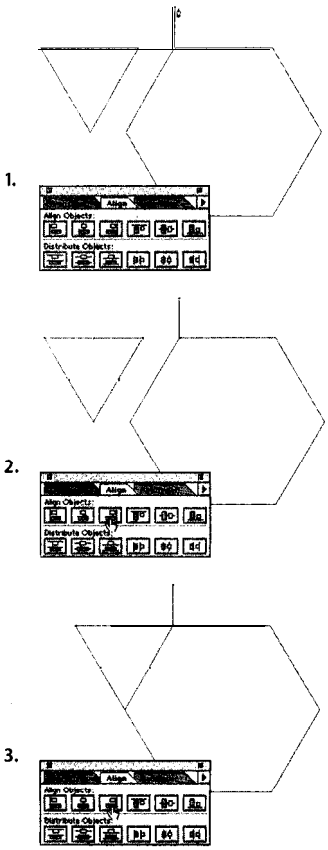
terns shown in this figure, we can come up with combinations of regular polygons that tessellate a plane. In this method, every point (or vertex) is surrounded by the identical arrangement of polygons. These patterns are called *semiregular tessellations* (as shown in figure 2 on page 45).

Regular and semiregular tessellations are often called "classical," "homogenous," or (my favorite, as he was an early hero of mine) "Archimedean" tessellations. Beyond these, there are an infinite number of *irregular tessellations*, in which there are different arrangements of regular polygons at different points.

How many possible combinations of regular polygons can we use to fill the space around a point? As it turns out, there are *21 (see* figure 1 on page 45). I've omitted parts of very large polygons-the numbers next to the sides indicate the number of sides in the polygon.

What about irregular *shapes?* Any nonequilateral triangle or quadrilateral will tessellate a plane, regardless of the lengths of its sides. Some irregular pentagons will also tessellate a plane, but not all of them will.

Impossible alignment. How can you align objects when there's no vertical edge to align the object to? In this example, I want to align one side of a triangle to one side of a hexagon in Illustrator.



1. With Snap To Point turned on, I draw a vertical line from one of the polygon's vertices (to do this, click the pen tool on the vertex, and then hold down Shift and click another point above it). In Illustrator 8.0, first turn on Disable Auto Add/Delete under General Preferences. Otherwise, when you draw the straight line from the corner of the polygon, you'll instead delete one of its anchor points.
2. Next, I select both the triangle and the path I've just drawn ...
3. ... and click the Align Right button. Illustrator aligns the right edge of the triangle with the path.At this point, I can delete the path or use it to help align other polygons.

Beyond the checkerboard

If these specific types of tessellation include all of the available options, then how did Escher create his complex, interlocking menageries of planarians and other animals? It's not nearly as complicated as it looks. He did it by distorting the sides of tessellating polygons, usually starting with one of the three regular tessellations. In some cases, he'd alter every side of the polygon in the same way; in other cases, he'd alter pairs of sides (generally sides opposite each other). Take a look at "Creating an Escher-style tessellation" on page 44. And then, when you're feeling brave, you can try "Making birds" on page 45.

Bibliography and resources

If you're interested in the geometry of tessellations, you should look at The Geometer's Sketchpad, a software program used mainly for teaching high school geometry It's a kind of cross between a sophisticated calculator and a drawing program. Demo versions are available from *www.keypress.com.*

The (almost literally) 800-pound gorilla of books on the topic is *Tilings and Patterns* by B. Griinbaum and G.C. Shephard (W. H. Freeman & Company, 1997). This very comprehensive treatment gives me inspiration every time I open it.

Introduction to Tessellations by Dale Seymour and Jill Britton (Dale Seymour Publications, 1989) is the book that first showed **me** Escher's methods in detail-if you liked this column, you'll love this book.

One of my favorites is *Islamic Patterns: An Analytical and Cosmological Approach* by Keith Critchlow (Thames and Hudson, 1976). It's out of print, but it's worth some searching in used bookstores.

There are many books on the art of M. C. Escher-one of the most recent is *The Graphic Work of M. C. Escher* (Wings Books, 1996).

But is it art?

I hope you've enjoyed this small adventure into geometry. Whenever I look at geometric art, I have the feeling that it was *discovered* rather than *created-as* if the pattern had always been there, and had just been waiting for someone to stumble across it.

This is, I imagine, the same feeling M. C. Escher had as he strung together his mind-boggling arrangements of horsemen, bats, smoke-snorting lizards, and flat-faced flounder. His work is unlike most other art-it's about human intellect and discovery rather than human emotion and communication. Whatever its appeal, I find it as thrilling and fascinating today as it was more than 35 years ago.

Thanks, Dad! ●

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