CS 404-31 - Analysis of Algorithms

Final Exam

December 3, 2003

1. Use Binary Search Recursion to search for the integer 26 in the following list of integers. Show the actions step by step.

   15 26 39 41 44 50 55 60

2. Suppose that there are $n = 2^k$ teams in an elimination tournament, in which there are $n/2$ games in the first round, with the $n/2 = 2^{k-1}$ winners playing in the second round and so on.

   (a) Develop a recurrence equation for the number of rounds in the tournament

   (b) How many rounds are there in the tournament when there are 256 teams?

   (c) Solve the recurrence equation of part (a).

3. Sort the following list showing the actions step by step, by using

   (a) Mergesort

   (b) Quicksort

252 176 315 121 343 276 122 305

4. Write an algorithm that prints out all the subsets of four elements of a set of n elements. The elements of this set are stored in a list that is the input to the algorithm. Define its basic operations and study its performance. Is that performance an every-case time complexity? Otherwise determine the worst-case complexity.

5. Use the algorithm for large integer multiplication (treshold 2) to find the product of 201299 and 15802. How many multiplications are needed to find this product?

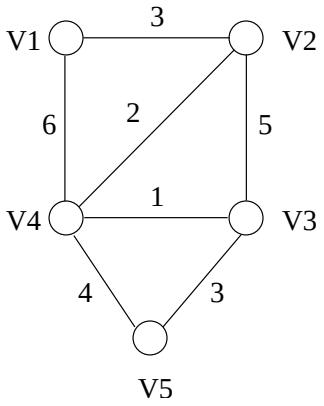
6. The following path matrix P is passed back to you from Floyd's algorithm:

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
v_1	0	9	5	0	0	9	9	9	5
v_2	0	0	5	0	1	0	9	9	0
v_3	9	9	0	9	9	9	9	9	0
v_4	0	0	5	0	0	0	0	7	0
v_5	0	9	0	9	0	9	9	9	3
v_6	0	9	5	0	1	0	9	9	0
v_7	8	8	8	0	8	8	0	0	8
v_8	6	0	0	2	6	0	9	0	3
v_9	8	8	8	7	8	8	0	7	0

What path from v_1 to v_9 has the minimum cost?

7. Find the minimum spanning tree for the following graph using:

   (a) Prim's Algorithm
   (b) Kruskal's Algorithm



8. Use Dijkstra's Algorithm to find the shortest path to all vertices from vertex v_4

9. Can you show a solution for the 4-queens problem where the queen on the first row is NOT placed on the second column? Show all your work.

10. Use the backtracking algorithm for the Sum-of-Subsets problem to find all combinations of the following numbers that sum to W = 52:

$$w_1 = 2 \quad w_2 = 10 \quad w_3 = 13 \quad w_4 = 17 \quad w_5 = 22 \quad w_6 = 42$$

11. Consider the following jobs and service times. Use a greedy algorithm to minimize the total amount of time spent in the system.

Job	*Service Time*
1	5
2	10
3	7
4	3

12. Use Huffman's algorithm to construct an optimal binary prefix code for the letters in the following table.

Letter	:	C	E	I	R	S	T	X
Frequency	:	11	22	16	12	15	10	14