

**CS 310T-31 - TPCS: Theory of Computation**  
**Final Exam**  
June 5, 2004

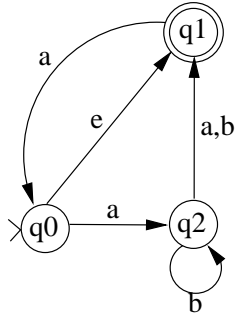
1. Construct deterministic finite automata accepting each of the following languages

(a)  $\{ w \in \{a, b\}^* : \text{each } a \text{ is immediately followed by three } b\text{'s} \}$

(b)  $\{ w \in \{0, 1\}^* : w \text{ has a number of } 1\text{'s that is multiple of three} \}$

2. What language is represented by the regular expression  $(a^*ba^*ba^*ba^*ba^*)^*$ ?

3. Given the following NFA, convert it to a DFA



4. Use the pumping lemma to show that the language  $B = \{0^n 1^n 2^n \mid n \geq 0\}$  is not context free

5. Construct a PDA that accepts the language  $\{a^n b^n \mid n \geq 0\}$   
(Extra Credit) Show that  $aba$  and  $baa \notin L(M)$  but  $aabb$  and  $ab \in L(M)$

6. Draw a parse tree for the following grammar and example:

$G = (W, \Sigma, R, S)$ , where

$W = \{S, (, )\}$ ,

$\Sigma = \{(, )\}$ ,

$R = \{S \rightarrow e, S \rightarrow SS, S \rightarrow (S)\}$

Example:  $(((((())())(())())))$

7. Give a Turing machine that scans a word to the right and accepts when it finds the sequence  $aba$  and rejects when it finds the sequence  $bab$ . In all the other cases the machine will not make a decision.

8. Give a state diagram of a turing machine that decide the following language:  $\{a^i b^j c^k \mid i, j, k \geq 0, j = 2i \text{ and } k = 2j\}$

(Extra Credit) Give the sequence of configurations that this machine enters when started on the following input strings

i.  $abbcccc$

ii.  $aaaabbc$

9. Let  $B$  be the set of all infinite sequences over  $\{a, b, c\}$ . Show that  $B$  is uncountable using a proof by diagonalization.

10. Let  $T = \{(i, j) \mid i, j \in \mathcal{N}\}$ . Show that  $T$  is countable

11. (Extra Credit) Suppose you have to find a phone number in a phone directory book. How much, using big 'O' notation will take you to find it, if you don't have a 'smart' way of searching? Do you know any better way of doing it? What's the efficiency of this better way?