

CS 310T-31 - TPCS: Theory of Computation

Final Exam

September 20, 2004

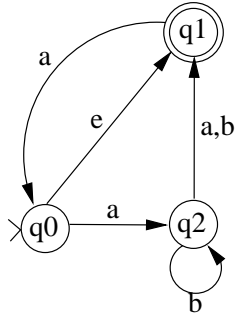
1. Construct deterministic finite automata accepting each of the following languages

(a) $\{ w \in \{a, b\}^* : \text{each } a \text{ is immediately followed by two } b\text{'s} \}$

(b) $\{ w \in \{0, 1\}^* : w \text{ has a number of } 1\text{'s that is multiple of five} \}$

2. What language is represented by the regular expression $(a^*bbba^*)^*$?

3. Given the following NFA, convert it to a DFA



4. Use the pumping lemma to show that the language $B = \{0^n 1^n 2^n \mid n \geq 0\}$ is not context free

5. Construct a PDA that accepts the language $\{a^n b^n \mid n \geq 0\}$
(Extra Credit) Show that aba and $baa \notin L(M)$ but $aabb$ and $ab \in L(M)$

6. Draw a parse tree for the following grammar and example:

$G = (W, \Sigma, R, S)$, where

$W = \{S, (,)\}$,

$\Sigma = \{(,)\}$,

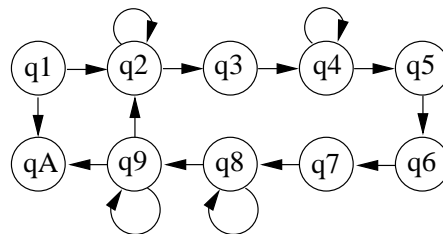
$R = \{S \rightarrow e, S \rightarrow SS, S \rightarrow (S)\}$

Example: $(((((())())(())())))$

7. Describe the actions of the following Turing Machine. Can you describe the language that the machine accepts? Can you show some words that are accepted and some words that are rejected by that language?

8. Complete the state diagram of a turing machine that decide the following language:
 $\{a^i b^j c^k \mid i, j, k \geq 0, j = 2i \text{ and } k = 2j\}$
 (Extra Credit) Give the sequence of configurations that this machine enters when started on the following input strings

- i. *abbcccc*
- ii. *aaaabbc*



9. Construct context-free grammars that generate the following languages:

(a) $\{ ww^R : w \in \{a, b\}^* \}$

(b) $\{ w \in \{a, b\}^* : w \text{ has twice as many } a\text{'s as } b\text{'s} \}$

(Extra Credit: Which strings can be produced by derivations of four or fewer steps for each one of the CFGs?)

10. Obtain a regular expression from the following finite automata. Simplify the regular expression as much as you can:

