

CS 310T-31 - TPCS: Theory of Computation

Final Exam

December 10, 2003

1. Draw state diagrams for each of the following languages

- (a) $\{ w \in \{a, b\}^* : w \text{ starts with } a \text{ and has an even length or starts with } b \text{ and has an odd length} \}$
- (b) $(a \cup b)^*aaa(a \cup b)^*$

2. Describe in English the following languages. (Extra Credit) Show 3 words that belong to each one of the languages.

- (a) $(a^*ba^*ba^*ba^*ba^*)^*$
- (b) $\{ w : \text{for some } u \in \Sigma\Sigma, w = uu^R u^R \} \ (\Sigma = \{a, b\})$
- (c) $L(M)$ generated by the pushdown automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where

$$\begin{aligned} K &= \{s, f\} \\ F &= \{f\} \\ \Sigma &= \{a, b, c\} \\ \Gamma &= \{a, b\} \\ \Delta &= \{((s, a, e), (s, aaa)), ((s, c, e), (f, e)), ((f, b, a), (f, e))\} \end{aligned}$$

3. Construct context-free grammars that generate the following languages:

(a) $\{ ww^R : w \in \{a, b\}^* \}$

(b) $\{ w \in \{a, b\}^* : w \text{ has twice as many } a\text{'s as } b\text{'s} \}$

(Extra Credit: Which strings can be produced by derivations of four or fewer steps for each one of the CFGs?)

4. For each of the following languages state if it is a RL, a CFL or none. If a language is both RL and CFL should be noticed as RL. Justify your answers.

(a) $\{ a^k b^k c^k d^k : k \geq 1 \}$

(b) $\{ w \in \{a, b\}^* : w \text{ has four as many } b\text{'s as } a\text{'s} \}$

(c) $\{ w \in \{a, b\}^* : \text{the total number of } a\text{'s is a multiple of four} \}$

(d) $\{ w \in \{a, b\}^* : w \text{ has a prime number of both } a\text{'s and } b\text{'s} \}$

5. Construct a PDA that accepts the language $\{w \in \{a, b, c\}^*, u \in \{a, b\}^* : w = udu^R\}$

6. Draw a parse tree for the following grammar and examples:

$G = (W, \Sigma, R, S)$, where

$W = \{S, T, F, a\}$,

$\Sigma = \{a, b\}$,

$R = \{S \rightarrow S + T, S \rightarrow T, T \rightarrow T * F, T \rightarrow F, F \rightarrow a, F \rightarrow b\}$

(a) $a + a * b$

(b) $a * b + b * b + b * a$

7. Give a Turing machine that scans a word to the right halts with a y when it finds three consecutive a 's and halts with an n when it finds two consecutive b 's

8. (Extra Credit) Obtain a regular expression from the following finite automata. Simplify the regular expression as much as you can:

