

CS 310T-31 - TPCS: Theory of Computation

Midterm 2

November 24, 2003

1. Construct context-free grammars that generate the following languages:

(a)  $\{ wdw^R : w \in \{a, b\}^* \}$

(b)  $\{ w \in \{a, b\}^* : w \text{ has twice as many } b\text{'s as } a\text{'s} \}$

(Extra Credit: Which strings can be produced by derivations of four or fewer steps for each one of the CFGs?)

2. Consider the pushdown automaton  $M = (K, \Sigma, \Gamma, \Delta, s, F)$ , where

$$K = \{s, f\}$$

$$F = \{f\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{a, b\}$$

$$\Delta = \{((s, a, e), (s, a)), ((s, b, e), (s, b)), ((s, c, e), (f, e)), ((f, a, a), (f, e)), ((f, b, b), (f, e))\}$$

(a) Show that  $abc$ ,  $abcab$ , and  $bca \notin L(M)$  but  $aca$ ,  $abcba$ , and  $bacab \in L(M)$

(b) Describe  $L(M)$  in English

3. Construct a PDA that accepts the language  $\{w \in \{a, b\}^* : w = w^R\}$

4. Draw a parse tree for the following grammar and example:

$G = (W, \Sigma, R, S)$ , where

$W = \{S, (, )\}$ ,

$\Sigma = \{(, )\}$ ,

$R = \{S \rightarrow e, S \rightarrow SS, S \rightarrow (S)\}$

Example:  $((()((()())))$

5. Give a Turing machine that scans a word to the right halts with a  $y$  when it finds two consecutive  $a$ 's and halts with an  $n$  when it finds three consecutive  $b$ 's

6. Which of the following languages are context-free? Explain briefly in each case.

(a)  $\{w \in \{a, b\}^* : w \text{ has four as many } b\text{'s as } a\text{'s}\}$

(b)  $\{w \in \{a, b\}^* : w \text{ has a prime number of } a\text{'s}\}$

(c)  $\{w \in \{a, b\}^* : w \text{ has a prime number of } b\text{'s}\}$

(d)  $\{w \in \{a, b\}^* : w \text{ has a prime number of both } a\text{'s and } b\text{'s}\}$

7. Let  $M = (K, \Sigma, \delta, s, \{h\})$ , where

$$\begin{aligned} K &= \{q_0, q_1, q_2, h\} \\ \Sigma &= \{a, b, -, >\} \\ s &= q_0, \end{aligned}$$

and  $\delta$  is given by the following table.

$q$	$\sigma$	$\delta(q, \sigma)$
$q_0$	$a$	$(q_1, \leftarrow)$
$q_0$	$b$	$(q_0, \rightarrow)$
$q_0$	$-$	$(q_0, \rightarrow)$
$q_0$	$>$	$(q_0, \rightarrow)$
$q_1$	$a$	$(q_1, \leftarrow)$
$q_1$	$b$	$(q_2, \rightarrow)$
$q_1$	$-$	$(q_1, \leftarrow)$
$q_1$	$>$	$(q_1, \rightarrow)$
$q_2$	$a$	$(q_2, \rightarrow)$
$q_2$	$b$	$(q_2, \rightarrow)$
$q_2$	$-$	$(h, -)$
$q_2$	$>$	$(q_2, \rightarrow)$

- Trace the computation of  $M$  starting from the configuration  $(q_0, > \underline{a}bb\_bb\_ \_\_\_aba)$ .
- Describe informally what  $M$  does when started in  $q_0$  on any square of a tape.
- (Extra Credit) Can you find a case where the machine will get into a “deadlock” (ie. get on an infinite loop of repeating the same set of operations)