CS 310T-31 - TPCS: Theory of Computation

Midterm 2

November 24, 2003

1. Construct context-free grammars that generate the following languages:

   (a) $\{ wdw^R : w \in \{a,b\}^* \}$

   (b) $\{ w \in \{a,b\}^* : w$ has twice as many b's as a's $\}$

   (Extra Credit: Which strings can be produced by derivations of four or fewer steps for each one of the CFGs?)

2. Consider the pushdown automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where

$$\begin{array}{lcl} K & = & \{s, f\} \\ F & = & \{f\} \\ \Sigma & = & \{a, b, c\} \\ \Gamma & = & \{a, b\} \\ \Delta & = & \{((s,a,e),(s,a)),((s,b,e),(s,b)),((s,c,e),(f,e)), \\ & & ((f,a,a),(f,e)),((f,b,b),(f,e))\} \end{array}$$

   (a) Show that abc, $abcab$, and $bca \notin L(M)$ but aca, $abcba$, and $bacab \in L(M)$

   (b) Describe $L(M)$ in English

3. Construct a PDA that accepts the language $\{w \in \{a,b\}^* : w = w^R\}$

4. Draw a parse tree for the following grammar and example:
   $G = (W, \Sigma, R, S)$, where
   $W = \{S, (,)\}$,
   $\Sigma = \{(,)\}$,
   $R = \{S \to e, \quad S \to SS, \quad S \to (S)\}$
   Example: $(()()(()()))$

5. Give a Turing machine that scans a word to the right halts with a y when it finds two consecutive a's and halts with an n when it finds three consecutive b's

6. Which of the following languages are context-free? Explain briefly in each case.

   (a) $\{w \in \{a,b\}^* : w \text{ has four as many } b'\text{s as } a'\text{s}\}$

   (b) $\{w \in \{a,b\}^* : w \text{ has a prime number of } a'\text{s}\}$

   (c) $\{w \in \{a,b\}^* : w \text{ has a prime number of } b'\text{s}\}$

   (d) $\{w \in \{a,b\}^* : w \text{ has a prime number of both } a'\text{s and } b'\text{s}\}$

7. Let $M = (K, \Sigma, \delta, s, \{h\})$, where

$$\begin{aligned} K &= \{q_0, q_1, q_2, h\} \\ \Sigma &= \{a, b, \_, >\} \\ s &= q_0, \end{aligned}$$

and δ is given by the following table.

q,	σ	$\delta(q, \sigma)$
q_0	a	$(q_1, \leftarrow)$
q_0	b	$(q_0, \rightarrow)$
q_0	_	$(q_0, \rightarrow)$
q_0	$>$	$(q_0, \rightarrow)$
q_1	a	$(q_1, \leftarrow)$
q_1	b	$(q_2, \rightarrow)$
q_1	_	$(q_1, \leftarrow)$
q_1	$>$	$(q_1, \rightarrow)$
q_2	a	$(q_2, \rightarrow)$
q_2	b	$(q_2, \rightarrow)$
q_2	_	$(h, \_)$
q_2	$>$	$(q_2, \rightarrow)$

(a) Trace the computation of M starting from the configuration $(q_0, > a\underline{b}b\_bb\_\_\_aba)$.

(b) Describe informally what M does when started in q_0 on any square of a tape.

(c) (Extra Credit) Can you find a case where the machine will get into a "deadlock" (ie. get on an infinite loop of repeating the same set of operations)