Using Chess to Teach Advanced Math Concepts

8 Queens Problem

- Problem: Put 8 queens on the chessboard so that none of the queens attack each other.
- First proposed in 1848 by Max Bezzel.
- Later generalized by Gauss and Cantor.
- Edsger Dijkstra used this problem in 1972 to illustrate structured programming.

Step 1

- There are 64 nCr 8 or 4.426 x 10^9 possible combinations of putting 8 queens on the 64 squares.
- Obviously, brute forcing this method could be quite painful, especially considering that the queen attacks anywhere from 28 to 35 squares depending on where it is on the board.
- There are a total of ~ 1.2 x 10^12 computations that need to be checked.

Part 2 Backtracking Depth First Algorithms

- Adding limitations to make the programming more efficient.
- Obviously, if there are 2 queens on the same rank or file, then it is not a solution.
- One step could be added into the program that says if there are two queens on the same rank, file, or diagonal, to go to the next possible solution.

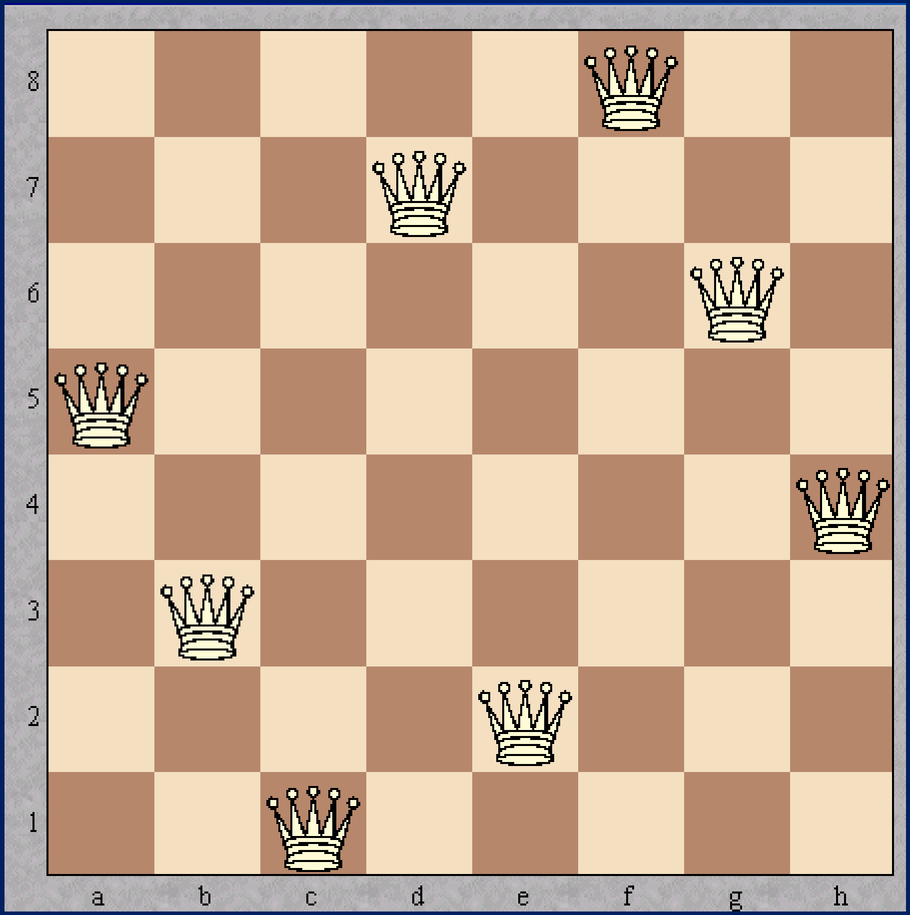
Part 3

- One programming level would include adding “knowledge into the program.”
- This is an extension of part 2: Filter out possible solutions where there is not one queen on each file and one on each rank.
- All of a sudden, there is a significantly smaller number of “possible solutions” to check.
- The number goes down to 8! = 40320.

Part 4

- There are 92 possible solutions, but only 12 distinct solutions (due to symmetry of the board).

One Possible Solution



Rotational Symmetry: The 12 Knights Problem

- This is a rotational geometry problem.
- This means that no matter what side of the chess board is facing you, the board will look the same.
- The object of the 12 knights problem is to have the 12 knights be on or attacking every square on the chessboard.

Step 1

- There has to be a knight posted to attack the two squares vertically and horizontally of each corner squares (a1, a8, h1, h8).
- The squares that need to be attacked are (a2, b1, a7, b8, h2, g1, g8, and h7).
- Therefore, there needs to be knights on c3, c6, f3, and f6.

Step 2

- There has to be a knight attacking the corner squares: a1, a8, h1, and h8.
- However, it has to be rotationally symmetrical.
- Therefore, the knights will go on c2, f7, g3, and b6. Obviously, the knights can also go onto c7, b3, f2, and g6. Same idea, different squares. This demonstrates the symmetry of the board along the diagonal.

Final step

- There are 4 “dead zone” squares which need to be attacked: b2, b7, g2, and g7.
- These are called dead zone squares because even though there are two knights by these squares these squares are not being attacked.
- The key is putting knights down on these squares while maintaining rotational symmetry.

Number of rectangles

- Problem: Prove that the number of rectangles in a nxn board is a perfect square.

Part 1

- 2x2 board: 4 1x1 rectangles, 2 2x1 rectangles, 2 1x2 rectangles, and 1 2x2 rectangle.
- 4+2+2+1 = 9
- 3x3 board: 9 1x1 rectangles, 6 1x2 rectangles, 3 1x3 rectangles, 6 2x1 rectangles, 4 2x2 rectangles, 2 2x3 rectangles, 3 3x1 rectangles, 2 3x2 rectangles, and 1 3x3 rectangle.
- 9+6+3+6+4+2+3+2+1 = 36

Part 2

- Data collected
- 1x1 board = 1 rectangle
- 2x2 board = 9 rectangles
- 3x3 board = 36 rectangles
- Now lets put this into a formula

Part 3

- $n = 1, 1 = 1^2$
- $n = 2, 9 = 3^2$
- $n = 3, 36 = 6^2$
- Here, we can see that the number of rectangles are squares, but we still cannot generalize for a n board.

Part 4

- $n = 1, 1 = 1^3$
- $n = 2, 9 = 1^3 + 2^3$
- $n = 3, 36 = 1^3 + 2^3 + 3^3$
- Another pattern that should be picked up on is that the number of rectangles is the sum of the cubes.

Part 5

- From arithmetic, we can quickly establish the following formula (if it is not already known):
- $1^3 + 2^3 + \ldots + n^3 = (1+2+\ldots+n)^2$
- The sum numbers is $\frac{n(n+1)}{2}$
- Therefore, we can conclude that the formula is $\left(\frac{n(n+1)}{2}\right)^2$

Questions?