

NEIU MATH304: Numerical Analysis, Demo Final Exam

Directions: Answer all of the questions. The problems will have their weights assigned next to them. Show all work for maximal credit. Good luck. No Books, Notes, or Communications Devices. Graphing and Scientific calculators are permitted. A 8 ½ by 11 sheet of paper with handwritten note is permitted. You have 110 minutes to complete the following questions. Good luck.

- 1) Develop an equation to estimate the fourth order derivative of $f(x)$ using a Taylor series expansion and the following points: $(x+3h)$, $(x+2h)$, x , $(x-2h)$, and $(x-3h)$. Accuracy is expected to be $O(h^2)$.
- 2) Discuss splines: cubic and bezier:
 - a. When are they used? What are they good at?
 - b. What data elements does they require / assume?
 - c. What errors / issues may be encountered for each set?
 - d. If you needed to use splines to plot a sine wave, discuss the benefits of one over the other. Illustrate as needed.
- 3) Discuss the Jacobi method and PA-LU as systems of equation solvers:
 - a. When is each used? What are each of them good at? Describe instances where one method is preferred over the other.
 - b. What data elements does it require / assume?
 - c. What differences do they have? Discuss in terms of error, convergence and stability.
- 4) Approximating The Golden ratio.
 - a. The golden ratio is $\frac{1+\sqrt{5}}{2}$. It is one of the solutions given by: $x^2 - x - 1 = 0$.
 - b. Isolate the linear term and re-configure it to solve for x .
 - c. Determine the convergence rate.
 - d. Using a starting point of $x_0=1$, find, x_3 . How close is this to our error estimate? How close is this to the golden ratio?
- 5) Runge-Kutta Method of order four (RK4):

The general algorithm is as follows:

$$w_{i+1} = w_i + \frac{h}{6}(s_1 + 2s_2 + 2s_3 + s_4)$$

where:

$$s_1 = f(t_i, w_i)$$

$$s_2 = f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}s_1\right)$$

$$s_3 = f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}s_2\right)$$

$$s_4 = f(t_i + h, w_i + hs_3)$$

- a) Define the variables that are used above.
- b) Write pseudocode that will run the above method. Choose appropriate inputs for your function and be sure to state what the inputs are.