NEIU MATH251: Discrete Mathematics, Demo Final Exam

Directions: **Answer all questions**. The problems will have their weights assigned next to them. Show all work for maximal credit. Good luck. No Books, Notes, Communications Devices, or Calculators. Scientific calculators are permitted. A single handwritten 8½ x 11 sheet of paper with notes. You have 110 minutes to complete the following questions. Good luck.

1) Definitions. Define the following:
   a. Define what a statement is. Provide an example.
   b. Describe a truth table. Provide an example.
   c. Define what an even number is. Provide an example.
   d. Define what divisibility is. Provide an example.
   e. Define what a rational number is. Provide an example.
   f. Define what a set is. Provide an example.
   g. Define disjoint. Provide an example.
   h. Define a partition. Provide an example.
   i. Define what Σ is. Provide an example of how it is used.
   j. Describe the differences between a permutation and a combination.
   k. Explain the "tree" and "slot" counting systems.
   l. What does it mean then a function is invertible?
   m. Describe the pigeon hole principle. Provide an example.
2) Prove or disprove: For statements A, B, and C: (do this twice.. use a truth table.. and prove using algebraic logic). $B \vee (\neg A \wedge \neg C) = \neg((\neg B \wedge C) \vee (\neg B \wedge A))$
3) Prove or disprove: The function $f(x) = \sin(x^2)$ is invertible for the domain of all positive real numbers between 0 and $\frac{\pi}{2}$, and the range of all positive real numbers.
4) Prove or disprove: $\forall x \in \mathbb{R}, |x|^2 \leq \lceil x \rceil^2$
5) Prove or disprove: Set of all prime numbers and the set of all composite numbers are partitions of the integer set.
6) Prove or disprove: Set of all prime numbers and the set of all composite numbers are partitions of the integer set, for all integers equal or greater than 2.
7) Prove or disprove: In any set of four consecutive integers, at least one of the integers is divisible by 4.
8) Prove or disprove: For the recursive relation given by:

   $a_n = a_{n-1} + 2a_{n-2}$ for all n $\geq$ 2 and with $a_0=1$ and $a_1=1$

   The direct formula is given by: $\forall n \geq 0, a_n = \frac{2}{3}2^n + \frac{1}{3}(-1)^n$ for all n $\geq$ 0.
9) Prove or disprove: the odds of getting a score of "21" in a game of blackjack on the opening deal is $\frac{80}{663}$.
10) Prove or disprove: $1{\bullet}2{\bullet}3 + 2{\bullet}3{\bullet}4 + 3{\bullet}4{\bullet}5 + ... + n{\bullet}(n+1){\bullet}(n+2) = 3!\binom{n+3}{1+3}$, for $\forall n \in \mathbb{Z}, n \geq 0$. (Hint: use summation notation)
11) Prove or disprove: $\frac{4x-3}{x+4}$ is a rational number given some $x \in \mathbb{R}$ such that $\frac{x+1}{5x-1}$ is a rational number.
12) Prove or disprove: e is an irrational number. (Hint: use a MacLaurin series expansion to help.. if you get really stuck.. try to Google this and maybe that will help)